Worksheet 3 Answer Key

1(a) $f(x) = \sin\left(\cos^6\left(\frac{9}{x^8}\right)\right) \overset{\text{prep}}{=} \underset{1}{\sin}\left(\left[\underset{3}{\cos}\left(\underset{4}{\frac{9}{x^8}}\right)\right]^{6\;\,2}\right)$ $\quad 9x^{-8}$

$$f'(x) = \boxed{\cos\left(\cos^6\left(\frac{9}{x^8}\right)\right)\cdot 6\left(\cos\left(\frac{9}{x^8}\right)\right)^5\cdot\left[-\sin\left(\frac{9}{x^8}\right)\right]\cdot\left(-72x^{-9}\right)}$$

1(b) $y = \tan\left(\frac{9}{\sin x}\right) \overset{\text{prep}}{=} \underset{1}{\tan}\left[9\,\underset{3}{(\sin x)}^{\overset{2}{-1}}\right]$

$$y' = \boxed{\sec^2\left(\frac{9}{\sin x}\right)\cdot 9\left[-(\sin x)^{-2}\right]\cdot\cos x}$$

OR Chain Rule + Quotient Rule

$$y' = \boxed{\sec^2\left(\frac{9}{\sin x}\right)\cdot\left(\frac{\sin x\cdot 0 - 9\cdot\cos x}{\sin^2 x}\right)}$$

Match. o.k.

1(c) $g(t) = \frac{8+\sec^2(7t)}{9+\cos t} \overset{\text{Prep}}{=} \frac{8+(\sec(7t))^2}{9+\cos t}$

$$g'(t) = \boxed{\frac{(9+\cos t)\cdot 2\left(\sec(7t)\right)^1\cdot\sec(7t)\tan(7t)\cdot 7 - \left(8+\sec^2(7t)\right)\cdot(-\sin t)}{(9+\cos t)^2}}$$

2. $f(x) = \overset{\text{product}}{\tan x\cdot\sin x} - \frac{1}{2}\overset{\text{chain}}{\sin(2x)}$

$f'(x) = \tan x\cdot\cos x + \sin x\cdot\sec^2 x - \frac{1}{\cancel{2}}\cos(2x)\cdot\cancel{2}$

$= \tan x\cdot\cos x + \sin x\cdot\sec^2 x - \cos(2x)$

$f'\left(\frac{\pi}{6}\right) = \overset{\frac{1}{\sqrt3}}{\tan\left(\frac{\pi}{6}\right)}\cdot\overset{\frac{\sqrt3}{2}}{\cos\left(\frac{\pi}{6}\right)} + \overset{\frac{1}{2}}{\sin\left(\frac{\pi}{6}\right)}\cdot\left[\overset{\left(\frac{2}{\sqrt3}\right)^2}{\sec\left(\frac{\pi}{6}\right)}\right]^2 - \overset{\frac{1}{2}}{\cos\left(\overset{\frac{\pi}{3}}{2\cdot\frac{\pi}{6}}\right)}$

$\sec\frac{\pi}{6} = \frac{1}{\cos\frac{\pi}{6}} = \frac{1}{\left(\frac{\sqrt3}{2}\right)} = \frac{2}{\sqrt3}$

$= \frac{1}{\cancel{\sqrt3}}\cdot\frac{\cancel{\sqrt3}}{2} + \frac{1}{\cancel{2}}\cdot\frac{\overset{2}{\cancel{4}}}{3} - \frac{1}{2}$

$= \cancel{\frac{1}{2}} + \frac{2}{3} - \cancel{\frac{1}{2}} = \frac{2}{3}$

3. $\sin(x+y) = 2x - 2y$ point (π, π)

$$\frac{d}{dx}\left(\sin(x+y)\right) = \frac{d}{dx}\left(2x-2y\right)$$

$$\cos(x+y)\cdot\left(1+\frac{dy}{dx}\right) = 2 - 2\cdot\frac{dy}{dx}$$

$$\cos(x+y) + \cos(x+y)\frac{dy}{dx} = 2 - 2\frac{dy}{dx}$$ Isolate

$-\cos(x+y)$ $+2\frac{dy}{dx}$

$$\cos(x+y)\frac{dy}{dx} + 2\frac{dy}{dx} = 2 - \cos(x+y)$$ Factor off $\frac{dy}{dx}$

$$\left(\cos(x+y)+2\right)\frac{dy}{dx} = 2 - \cos(x+y)$$ Solve for $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{2-\cos(x+y)}{\cos(x+y)+2}$$

Specific Slope $$\left.\frac{dy}{dx}\right|_{(x,y)=(\pi,\pi)} = \frac{2-\cos(\overbrace{\pi+\pi}^{2\pi})}{\cos(\underbrace{\pi+\pi}_{2\pi})+2} = \frac{2-\cancel{\cos(2\pi)}^{1}}{\cancel{\cos(2\pi)}_{1}+2} = \frac{1}{3}$$

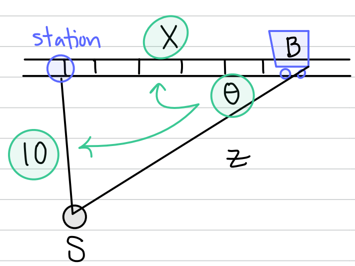
Tangent Line: Point-Slope Form

$$y-\pi = \frac{1}{3}(x-\pi)$$

$$y-\pi = \frac{1}{3}x - \frac{\pi}{3}$$
$+\pi$ $+\pi$

$$\boxed{y = \frac{x}{3} + \frac{2\pi}{3}}$$ Match!

4. Diagram



Variables

Let x = Distance between Train and Station

z = Distance between Bob (on Train) and Sally

θ = Angle of Bob's Head Rotation from Track Line

Given $\frac{d\theta}{dt} = -2$ Radians / second

Find $\frac{dx}{dt} = ?$ when $z = 13$ meters

Equation

$$\tan\theta = \frac{10}{x}$$

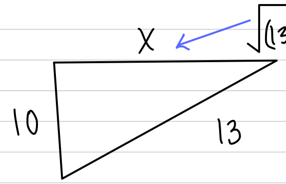
Differentiate

$$\frac{d}{dt}\left(\tan\theta\right) = \frac{d}{dt}\left(10x^{-1}\right)$$

$$\sec^2\theta \cdot \frac{d\theta}{dt} = -10x^{-2}\,\frac{dx}{dt}$$

$$\left(\sec\theta\right)^2 \frac{d\theta}{dt} = \frac{-10}{x^2} \cdot \frac{dx}{dt}$$

Extra Solvable Information



$$x = \sqrt{(13)^2 - (10)^2} = \sqrt{169 - 100} = \sqrt{69}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(\frac{A}{H}\right)} = \frac{H}{A} = \frac{13}{\sqrt{69}}$$

Substitute

$$\left(\frac{13}{\sqrt{69}}\right)^2 \cdot (-2) = -\frac{10}{\left(\sqrt{69}\right)^2} \cdot \frac{dx}{dt}$$

$$\frac{169}{69}(-2) = \frac{-10}{69} \cdot \frac{dx}{dt}$$

Solve

$$\frac{dx}{dt} = \frac{169}{\cancel{69}}\left(\cancel{-2}\right) \cdot \frac{\cancel{69}}{\cancel{-10}_{\,5}} = \frac{169}{5} \quad m/sec$$

Answer

$\boxed{\text{The train is travelling } \frac{169}{5} \text{ meters every second at that moment.}}$

5 (a) $f''(x) = 20x^3 + 12x^2 + 4$ $\qquad f(0) = 8$ and $f(1) = 5$

$$f'(x) = \int f''(x)\,dx = \int 20x^3 + 12x^2 + 4\,dx$$

constant

$$= \frac{\overset{5}{\cancel{20}}x^4}{\cancel{4}} + \frac{\overset{4}{\cancel{12}}x^3}{\cancel{3}} + 4x + C$$

note: no f' info given

$$f(x) = \int f'(x)\,dx = \int 5x^4 + 4x^3 + 4x + C$$

$$= \frac{\cancel{5}x^5}{\cancel{5}} + \frac{\cancel{4}x^4}{\cancel{4}} + \frac{\overset{2}{\cancel{4}}x^2}{\cancel{2}} + Cx + D$$

$$= x^5 + x^4 + 2x^2 + Cx + \cancel{D}^{\,8}$$

$$f(0) = \underbrace{0 + 0 + 0 + 0}_{\text{all } 0} + D \overset{\text{set}}{=} 8 \quad \Rightarrow \quad D = 8$$

$$f(1) = \underbrace{1 + 1 + 2}_{4} + C + 8 \overset{\text{Set}}{=} 5 \qquad C + 12 = 5 \Rightarrow C = -7$$

Finally,

$$f(x) = \boxed{x^5 + x^4 + 2x^2 - 7x + 8}$$

5(b) $a(t) = 3\cos t - 2\sin t$ $\qquad$ Given $s(0) = 0$, $v(0) = 4$

$$v(t) = \int a(t)\,dt = \int 3\cos t - 2\sin t\,dt$$

$$= 3\sin t + 2\cos t + \cancel{C_1}^{\,2}$$

Use Given Value

$$v(0) = 3\cancel{\sin 0}^{\,0} + 2\cancel{\cos 0}^{\,1} + C_1 \overset{\text{set}}{=} 4$$

$$0 + 2 + C_1 = 4 \Rightarrow C_1 = 2$$

Collect $v(t) = 3\sin t + 2\cos t + 2$

$$s(t) = \int v(t)\,dt = \int 3\sin t + 2\cos t + 2\,dt$$

$$= -3\cos t + 2\sin t + 2t + C_2$$

Use next given value

$$s(0) = -3\cancel{\cos 0}^{\,1} + 2\cancel{\sin 0}^{\,0} + 0 + C_2 \overset{\text{set}}{=} 0$$

$$-3 + C_2 = 0 \Rightarrow C_2 = 3$$

Collect $\boxed{s(t) = -3\cos t + 2\sin t + 2t + 3}$

split-split

$$6(a)\quad \int \frac{7x^{2/5}+8x^{-4/3}+\frac{1}{x}}{\sqrt{x}}\,dx = \int \frac{7x^{2/5}}{x^{1/2}} + \frac{8x^{-4/3}}{x^{1/2}} + \frac{x^{-1}}{x^{1/2}}\,dx$$

$x^{1/2}$

note: All Algebra Prep

$$= \int \frac{7x^{4/10}}{x^{5/10}} + \frac{8x^{-8/6}}{x^{3/6}} + \frac{x^{-2/2}}{x^{1/2}}\,dx$$

$$= \int 7x^{-1/10} + 8x^{-11/6} + x^{-3/2}\,dx$$

$$= \frac{10}{9}\ \frac{7x^{9/10}}{9/10} + \frac{-6}{5}\ \frac{8x^{-5/6}}{-5/6} + -2\ \frac{x^{-1/2}}{-1/2} + C$$

$$= \boxed{\frac{70}{9}x^{9/10} - \frac{48}{5}x^{-5/6} - 2x^{-1/2} + C}$$

or//

$$= \frac{70}{9}x^{9/10} - \frac{48}{5x^{5/6}} - \frac{2}{\sqrt{x}} + C$$

$$6(b)\quad \int\left(\sqrt{3}+\frac{1}{x^3}\right)\left(x-\frac{1}{x^{2/7}}\right)dx = \int \sqrt{3}\,x - \frac{\sqrt{3}}{x^{2/7}} + \frac{x}{x^3} - \frac{1}{x^3\cdot x^{2/7}}\,dx$$

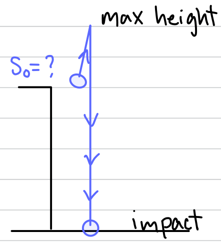
$x^3\cdot x^{2/7} = x^{21/7}x^{2/7} = x^{23/7}$

$$= \int \sqrt{3}\,x - \sqrt{3}\,x^{-2/7} + x^{-2} - x^{-23/7}\,dx$$

$$= \frac{\sqrt{3}\,x^2}{2} - \frac{\sqrt{3}\,x^{5/7}}{5/7} + \frac{x^{-1}}{-1} - \frac{x^{-16/7}}{-16/7} + C$$

$$= \boxed{\frac{\sqrt{3}\,x^2}{2} - \frac{7\sqrt{3}}{5}x^{5/7} - \frac{1}{x} + \frac{7}{16}x^{-16/7} + C}$$

7.



Equations of Motion

$$\begin{cases} a(t) = -32 \\ v(t) = -32t + \overset{80}{v_0} \\ s(t) = -16t^2 + \overset{80}{v_0}t + \overset{?}{S_0} \end{cases}$$

Given : $v(0) = \overset{+}{80}$ ft/sec

$v(t_{impact}) = -112$ ft/sec

Find : $S_0 = ?$

Impact: $v(t_{impact}) = -32t + 80 \overset{set}{=} -112$

$32t = 192$

$t_{impact} = \frac{192}{32} = 6$ seconds

Plug $t_{impact} = 6$ seconds into position and set equal to 0

That is, $s(t_{impact}) = s(6) = 0$

$$s(6) = -16(6)^2 + 80(6) + S_0 = 0$$

note: S_0 only unknown

$$-576 + 480 + S_0 = 0$$

$$-96 + S_0 = 0$$

$$\Rightarrow S_0 = 96 \text{ feet}$$

$$\begin{array}{r} \overset{3}{36} \\ \underline{16} \\ 216 \\ \underline{360} \\ 576 \end{array}$$

Answer: $\boxed{\text{The building is 96 feet tall}}$