Worksheet #2 Answer Key

1. $\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1} + C}$ $(n \neq -1)$

2. $\int \sqrt{x}\,dx = \int x^{1/2}\,dx = \frac{x^{3/2}}{3/2} + C = \frac{2}{3}x^{3/2} + C$

3. prep
$\int \frac{1}{x^9}\,dx = \int x^{-9}\,dx = \frac{x^{-8}}{-8} + C = -\frac{1}{x^8} + C$

4. prep
$\int \frac{1}{\sqrt{x}}\,dx = \int x^{-1/2}\,dx = \frac{x^{1/2}}{1/2} + C = 2\sqrt{x} + C$

5. prep
$\int \frac{1}{x^{3/7}}\,dx = \int x^{-\frac{3}{7}}\,dx = \frac{x^{4/7}}{4/7} + C = \frac{7}{4}x^{4/7} + C$

6. $\int \cos x\,dx = \sin x + C$

7. $\int \sin x\,dx = -\cos x + C$

8. $\int \sec^2 x\,dx = \tan x + C$

9. $\int \sec x \cdot \tan x\,dx = \sec x + C$

Constant

10. $f'(x) = \frac{d}{dx}\left(\frac{3}{4}x + x^{3/4} - \frac{1}{x^{3/4}} + \frac{4}{3} + \frac{1}{x^{4/3}} + \frac{3}{4x^4} - \frac{4}{x^3}\right)$

prep

$= \frac{d}{dx}\left(\frac{3}{4}x + x^{3/4} - x^{-3/4} + \frac{4}{3} + x^{-4/3} + \frac{3}{4}x^{-4} - 4x^{-3}\right)$

$= \frac{3}{4} + \frac{3}{4}x^{-1/4} + \frac{3}{4}x^{-7/4} + 0 - \frac{4}{3}x^{-7/3} + \cancel{\frac{3}{4}}(\cancel{-4}x^{-5}) + 12x^{-4}$

$= \frac{3}{4} + \frac{3}{4}x^{-1/4} + \frac{3}{4}x^{-7/4} - \frac{4}{3}x^{-7/3} - 3x^{-5} + 12x^{-4}$

OR $= \frac{3}{4} + \frac{3}{4x^{1/4}} + \frac{3}{4x^{7/4}} - \frac{4}{3x^{7/3}} - \frac{3}{x^5} + \frac{12}{x^4}$

11. $\int f(x)\,dx = \int \frac{3}{4}x + x^{3/4} - \frac{1}{x^{3/4}} + \frac{4}{3} + \frac{1}{x^{4/3}} + \frac{3}{4x^4} - \frac{4}{x^3}\,dx$

prep as above in #10 $\overset{\text{prep}}{=} \int \frac{3}{4}x + x^{3/4\,+1} - x^{-3/4\,+1} + \frac{4}{3} + x^{-4/3\,+1} + \frac{3}{4}x^{-4\,+1} - 4x^{-3\,+1}\,dx$

$= \frac{3}{4}\cdot\frac{x^2}{2} + \frac{x^{7/4}}{7/4} - \frac{x^{1/4}}{1/4} + \frac{4}{3}\cdot x + \frac{x^{-1/3}}{-1/3} + \frac{\cancel{3}}{4}\cdot\frac{x^{-3}}{-\cancel{3}} - \overset{2}{\cancel{4}}\cdot\frac{x^{-2}}{-\cancel{2}} + C$

Simplify $= \boxed{\frac{3x^2}{8} + \frac{4}{7}x^{7/4} - 4x^{1/4} + \frac{4}{3}x - 3x^{-1/3} - \frac{x^{-3}}{4} + 2x^{-2} + C}$

OR// $= \frac{3x^2}{8} + \frac{4}{7}x^{7/4} - 4x^{1/4} + \frac{4}{3}x - \frac{3}{x^{1/3}} - \frac{1}{4x^3} + \frac{2}{x^2} + C$

12. $\int x^3(1+x^2)\,dx = \int x^3 + x^5\,dx = \frac{x^4}{4} + \frac{x^6}{6} + C$

13. $\int \frac{x+\sqrt{x}+7}{x^3}\,dx = \int \frac{x}{x^3} + \frac{\sqrt{x}}{x^3} + \frac{7}{x^3}\,dx$ Split-Split ($x^{1/2}$, $6/2$)

OR// $\int (x + x^{1/2} + 7)\,x^{-3}\,dx$

⋮

prep $= \int x^{-2} + x^{-5/2} + 7x^{-3}\,dx$

$= \frac{x^{-1}}{-1} + \frac{x^{-3/2}}{-3/2} + \frac{7x^{-2}}{-2} + C$

$= -x^{-1} - \frac{2}{3}x^{-3/2} - \frac{7}{2}x^{-2} + C$

OR// $= -\frac{1}{x} - \frac{2}{3x^{3/2}} - \frac{7}{2x^2} + C$

14. $\int x^2 + x(1+x)^2\,dx = \int x^2 + x(1+x)(1+x)\,dx$ FOIL Algebra

$x+x$

$= \int x^2 + x(1+2x+x^2)\,dx$ Distribute

$= \int x^2 + x + 2x^2 + x^3\,dx$

Combine + Simplify

$= \int x^3 + 3x^2 + x\,dx$

$= \frac{x^4}{4} + \frac{\cancel{3}x^3}{\cancel{3}} + \frac{x^2}{2} + C$

$= \boxed{\frac{x^4}{4} + x^3 + \frac{x^2}{2} + C}$

15. $\int -3\cos x - \sec^2 x - 7\sec x\tan x - \sin x\,dx$

$+$

$= -3(\sin x) - \tan x - 7(\sec x) - (-\cos x) + C$

$= \boxed{-3\sin x - \tan x - 7\sec x + \cos x + C}$

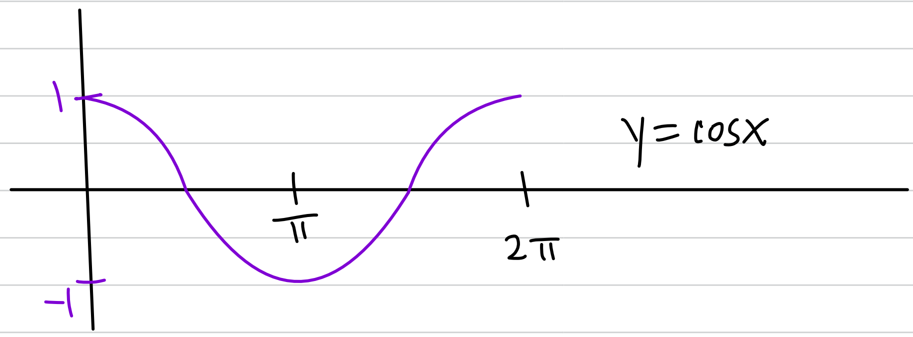
16. $y = 2x + \sin x$ Horizontal Tangent Line means Derivative equals 0

$y' = 2 + \cos x \overset{\text{set}}{=} 0$

Solve $\cos x = -2$ which has no solution

not possible → Recall Range of cosine is $[-1,1]$

OR, think about Unit Circle.



17. $f(x) = \int f'(x)\,dx = \int 2 + \sin x \, dx$

$$= 2x - \cos x + C$$ ← the Most General Antiderivative

Test Initial Value

$$f\left(\frac{\pi}{2}\right) = \cancel{2}\left(\frac{\pi}{\cancel{2}}\right) - \cancel{\cos\left(\frac{\pi}{2}\right)}^{0} + C \overset{\text{Set}}{=} 3$$

$$\pi + C = 3 \overset{\text{Solve}}{\Rightarrow} C = 3 - \pi$$ plug back in...

Finally, $f(x) = 2x - \cos x + 3 - \pi$

18. $f(x) = \int f'(x)\,dx = \int x^2 + 1 \, dx = \frac{x^3}{3} + x + C$

Test Initial Value

$$f(1) = \frac{1}{3} + 1 + C \overset{\text{set}}{=} 3$$

$$\frac{4}{3} + C = 3 \overset{\text{Solve}}{\Rightarrow} C = 3 - \frac{4}{3} = \frac{9}{3} - \frac{4}{3} = \frac{5}{3}$$ plug back in...

Finally, $f(x) = \frac{x^3}{3} + x + \frac{5}{3}$

19. $f(x) = \int f'(x)\,dx = \int x\left(2 + \sqrt{x}\right)dx = \int 2x + x^{3/2}\,dx$

$$= \frac{\cancel{2}x^2}{\cancel{2}} + \frac{x^{5/2}}{5/2} + C$$

$$= x^2 + \frac{2}{5}x^{5/2} + C$$

Test Initial Value

$$f(4) = 16 + \frac{2}{5}(4)^{5/2} + C \overset{\text{set}}{=} 30$$

$$16 + \frac{2}{5}\left(\cancel{\sqrt{4}}\right)^{5} + C = 30$$

$$16 + \frac{64}{5} + C = 30 \overset{\text{Solve}}{\Rightarrow} C = 30 - 16 - \frac{64}{5}$$

$$= 14 - \frac{64}{5}$$

$$= \frac{70}{5} - \frac{64}{5} = \frac{6}{5}$$ plug back in

Finally, $f(x) = x^2 + \frac{2}{5}x^{5/2} + \frac{6}{5}$

20. $f'(x) = \int f''(x)\,dx = \int \frac{1}{\sqrt{x}} + 3x^2\,dx$ prep $\to x^{-1/2}$

$= \frac{x^{1/2}}{1/2} + \frac{\cancel{3}x^3}{\cancel{3}} + C$

$= 2\sqrt{x} + x^3 + C_1$ use one f' condition to find C_1

Test Initial Value

$f'(1) = 2\cdot\cancel{\sqrt{1}}^{\,1} + 1 + C_1 \overset{\text{set}}{=} 2$

$\cancel{2} + 1 + C_1 = \cancel{2} \overset{\text{Solve}}{\Rightarrow} C_1 = -1$

Collect so far:

$f'(x) = 2\sqrt{x} + x^3 - 1$

Next:

$f(x) = \int f'(x)\,dx = \int 2x^{1/2} + x^3 - 1\,dx$

$= 2\cdot\frac{x^{3/2}}{3/2} + \frac{x^4}{4} - x + C_2$ use other f value to find C_2

$= \frac{4}{3}x^{3/2} + \frac{x^4}{4} - x + C_2$

Test Value

$f(1) = \frac{4}{3}\cancel{(1)}^{3/2} + \frac{1}{4} - 1 + C_2 \overset{\text{set}}{=} 0$

$\frac{4}{3} + \frac{1}{4} - 1 + C_2 = 0$

$\underbrace{\frac{16}{12} + \frac{3}{12} - \frac{12}{12}}_{\frac{7}{12}} + C_2 = 0 \Rightarrow C_2 = \frac{-7}{12}$

Finally, $f(x) = \frac{4}{3}x^{3/2} + \frac{x^4}{4} - x - \frac{7}{12}$

GUESS AND CHECK

21. $f(x) = \frac{-\cos(3x)}{3}$ because $\frac{d}{dx}\left(-\frac{1}{3}\cos(3x)\right) = \cancel{-}\frac{1}{\cancel{3}}\left(\cancel{-}\sin(3x)\cdot\cancel{3}\right) = \sin(3x)$ ✓

Guess and Check

22. $f(x) = 2\sqrt{\sec x + 8} + C = 2(\sec x + 8)^{1/2} + C$

because $f'(x) = \cancel{2} \cdot \left(\frac{1}{\cancel{2}\sqrt{\sec x+8}}\right) \cdot (\sec x \cdot \tan x) = \frac{\sec x \tan x}{\sqrt{\sec x + 8}}$ Match ✓

Now Test Value

$f(0) = 2\sqrt{(\cancel{\sec 0}) + 8} + C \overset{\text{set}}{=} 7$

$\sec 0 = \frac{1}{\cancel{\cos 0}} = 1$

1

$\cancel{\sqrt{9}} \to 3$

$6 + C = 7 \Rightarrow C = 1$

Finally. $f(x) = 2\sqrt{\sec x + 8} + 1$