

Worksheet #1 Answer Key

1.  $y = x^{\frac{3}{2}} \cdot \tan x$

$y' = x^{\frac{3}{2}} \cdot \sec^2 x + \tan x \cdot \frac{3}{2} x^{\frac{1}{2}}$

2.  $f(x) = x \cos x - \sin x$

$f'(x) = x(-\sin x) + \cos x \cdot (1) - \cos x = -x \sin x$

3.  $y = \sec^7 x = (\sec x)^7$

$y' = 7(\sec x)^6 \cdot (\sec x \cdot \tan x) = 7 \sec^7 x \cdot \tan x$

4.  $f(x) = \tan \sqrt{1-x^8}$

$f'(x) = \sec^2 \sqrt{1-x^8} \cdot \frac{1}{\sqrt{1-x^8}} \cdot (-8x^7) = \frac{-4x^7 \sec^2 \sqrt{1-x}}{\sqrt{1-x}}$

5.  $y = \cos\left(\frac{1}{x}\right) = \cos(x^{-1})$

$y' = -\sin\left(\frac{1}{x}\right) \cdot (-x^{-2}) = \frac{\sin\left(\frac{1}{x}\right)}{x^2}$

6.  $y = \frac{1}{\cos x} = (\cos x)^{-1}$

Chain Rule:

$y' = -(\cos x)^{-2} \cdot (-\sin x) = \frac{\sin x}{\cos^2 x}$

OR also equals

$\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = \sec x \cdot \tan x$

OR Quotient Rule:

$y' = \frac{\cos x(0) - 1(-\sin x)}{\cos^2 x} = \frac{\sin x}{\cos^2 x}$  Matches above

OR notice  $y = \frac{1}{\cos x} = \sec x$  by definition

$\hookrightarrow y' = \sec x \tan x$  equivalent

7.  $f(x) = \sin^6(x^3 - 5x) = (\sin(x^3 - 5x))^6$

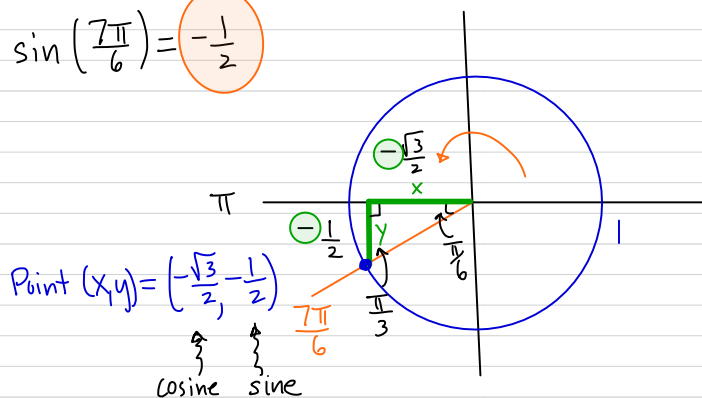
$f'(x) = 6(\sin(x^3 - 5x))^5 \cdot \cos(x^3 - 5x) \cdot (3x^2 - 5)$   
 $\sin^5(x^3 - 5x)$

$$8. y = \sqrt{\tan \sqrt{x}}$$

$$y' = \frac{1}{2\sqrt{\tan \sqrt{x}}} \cdot \sec^2 \sqrt{x} \cdot \frac{1}{2\sqrt{x}} = \frac{\sec^2 \sqrt{x}}{4\sqrt{\tan \sqrt{x}} \cdot \sqrt{x}}$$

could also combine roots

$$9. \sin\left(\frac{7\pi}{6}\right) = -\frac{1}{2}$$



$$10. H(x) = \sin(7x) \quad \rightarrow \quad H'(x) = \cos(7x) \cdot 7 = 7 \cos(7x)$$

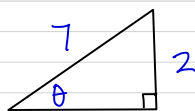
$$H'\left(\frac{\pi}{6}\right) = 7 \cos\left(\frac{7\pi}{6}\right) = 7 \left(-\frac{\sqrt{3}}{2}\right) = -\frac{7\sqrt{3}}{2}$$

x-coordinate

Note: from #9 above we showed the  $\cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

$$11. \sin \theta = \frac{2}{7} \quad \frac{O}{H}$$

$$\Rightarrow \tan \theta = \frac{2}{\sqrt{45}} = \frac{2}{3\sqrt{5}}$$



$$\Rightarrow \sqrt{7^2 - 2^2} = \sqrt{49 - 4} = \sqrt{45} = \sqrt{9 \cdot 5} = 3\sqrt{5}$$

$$\sec \theta = \frac{H}{A} = \frac{7}{\sqrt{45}} = \frac{7}{3\sqrt{5}}$$

Note: could also use Identities

$$12. f(x) = \tan(2x) + \cos(2x) + 16 \cos x$$

$$f'(x) = \sec^2(2x) \cdot 2 - \sin(2x) \cdot 2 - 16 \sin x$$

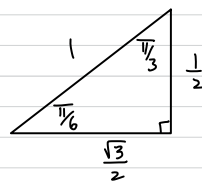
$$= 2 \sec^2(2x) - 2 \sin(2x) - 16 \sin x \quad \checkmark \text{ Match}$$

$$f'\left(\frac{\pi}{6}\right) = 2 \left[ \sec\left(2\left(\frac{\pi}{6}\right)\right) \right]^2 - 2 \sin\left(2\left(\frac{\pi}{6}\right)\right) - 16 \sin\left(\frac{\pi}{6}\right)$$

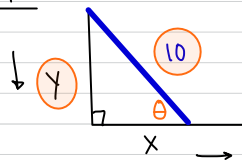
$$= 2 \left[ \sec\left(\frac{\pi}{3}\right) \right]^2 - 2 \sin\left(\frac{\pi}{3}\right) - 16 \sin\left(\frac{\pi}{6}\right)$$

$$= 2 \cdot 4 - 2 \left(\frac{\sqrt{3}}{2}\right) - 16 \left(\frac{1}{2}\right) = 8 - \sqrt{3} - 8 = -\sqrt{3} \quad \text{Match}$$

Note:  $\sec\left(\frac{\pi}{3}\right) = \frac{1}{\cos\left(\frac{\pi}{3}\right)} = \frac{1}{\frac{1}{2}} = 2$



13. Diagram:



Variables:

Let  $x$  = Distance from base of ladder to the wall

$y$  = Distance from top of ladder to the ground

Given  $\frac{dy}{dt} = -1$  ft/sec.

Find  $\frac{d\theta}{dt} = ?$  when  $y = 3$

Equation:

$$\sin\theta = \frac{y}{10}$$

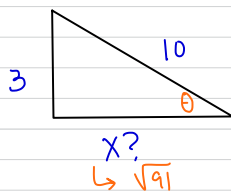
Differentiate:

$$\frac{d}{dt}(\sin\theta) = \frac{d}{dt}\left(\frac{y}{10}\right)$$

Note: Need  $x$ -value to solve for  $\cos\theta = \frac{A}{H}$

$$\cos\theta \cdot \frac{d\theta}{dt} = \frac{1}{10} \cdot \frac{dy}{dt}$$

Extra Solvable Information:



$$x^2 + 3^2 = (10)^2$$

$$x^2 = 100 - 9 = 91$$

$$x = \sqrt{91}$$

Substitute: Key Moment Info

$$\cos\theta \cdot \frac{d\theta}{dt} = \frac{1}{10} \cdot \frac{dy}{dt}$$

$$\frac{\sqrt{91}}{10} \cdot \frac{d\theta}{dt} = \frac{1}{10} \cdot (-1)$$

Solve:

$$\frac{d\theta}{dt} = -\frac{1}{10} \left(\frac{10}{\sqrt{91}}\right) = -\frac{1}{\sqrt{91}} \text{ Rad/sec.}$$

Answer:

The angle between the ladder and the ground is  
Decreasing at a rate of  $\frac{1}{\sqrt{91}}$  Radians per second.

Takes care of minus

#### 14. Implicit Differentiation

$$\frac{d}{dx}(y^2 + \cos x) = \frac{d}{dx}(x \cdot y^3)$$

$$2y \frac{dy}{dx} - \sin x = x \cdot 3y^2 \frac{dy}{dx} + y^3 \quad (1) \quad \text{Isolate}$$

$$2y \frac{dy}{dx} - 3xy^2 \frac{dy}{dx} = y^3 + \sin x \quad \text{Factor}$$

$$(2y - 3xy^2) \frac{dy}{dx} = y^3 + \sin x \quad \text{Solve}$$

$$\frac{dy}{dx} = \frac{y^3 + \sin x}{2y - 3xy^2}$$

#### 15. Challenge:

$$\frac{d}{dx} \cos \left( \frac{\frac{6}{x^6} + \tan(3x)}{\frac{4}{x} + \sec x} \right)$$

prep  
6x<sup>-6</sup>  
4x<sup>-1</sup>

$$= -\sin \left( \frac{\frac{6}{x^6} + \tan(3x)}{\frac{4}{x} + \sec x} \right) \cdot \left[ \frac{\left( \frac{4}{x} + \sec x \right) \left( -36x^{-7} + \sec^2(3x) \cdot 3 \right) - \left( \frac{6}{x^6} + \tan(3x) \right) \cdot \left( -4x^{-2} + \sec x \tan x \right)}{\left( \frac{4}{x} + \sec x \right)^2} \right]$$

Chain Rule  $\hookrightarrow$  Quotient Rule