Homework 16 Answer Key

Also WS #10

Spring 22

1. $y = x\ln x - x$

$y' = \cancel{x}\cdot\frac{1}{\cancel{x}} + \ln x\,(1) - 1 = \cancel{1} + \ln x - \cancel{1} = \ln x$

2. $y = \sin(\ln x)$

$y' = \cos(\ln x)\cdot\frac{1}{x} = \frac{\cos(\ln x)}{x}$

3. $y = \ln(\sin x)$

$y' = \frac{1}{\sin x}\cdot\cos x = \frac{\cos x}{\sin x}$

4. $f(x) = \ln\left(\frac{1}{x}\right)$

$f'(x) = \frac{1}{\frac{1}{x}}\cdot\left(-\frac{1}{x^2}\right) = \cancel{x}\left(-\frac{1}{x^{\cancel{2}}}\right) = -\frac{1}{x}$

OR, $f(x) = \ln\left(\frac{1}{x}\right) = \ln\left(x^{-1}\right) = -\ln x$

$f'(x) = -\frac{1}{x}$

Match!

Log Power Rule

$\ln(a^b) = b\ln a$

5. $f(x) = \frac{1}{\ln x} = (\ln x)^{-1}$

$f'(x) = -(\ln x)^{-2}\cdot\frac{1}{x} = -\frac{1}{x(\ln x)^2}$

6. $y = \ln(\ln(\ln x))$

$y' = \boxed{\frac{1}{\ln(\ln x)}\cdot\frac{1}{\ln x}\cdot\frac{1}{x}}$

7. $y = (\ln x)^3$

$y' = \boxed{3(\ln x)^2\cdot\frac{1}{x}}$

8. $y = \ln(x^3)$

$y' = \frac{1}{\cancel{x^3}_{x}}\cdot(3\cancel{x^2}) = \frac{3}{x}$

OR, $y = \ln(x^3) = 3\ln x$

$y' = 3\cdot\frac{1}{x} = \frac{3}{x}$

9. $y = \ln(e^{-3x})$

$y' = \frac{1}{\cancel{e^{-3x}}} \cdot \cancel{e^{-3x}} \cdot (-3) = -3$

OR// $y = \cancel{\ln}(\cancel{e}^{-3x}) = -3x$

$y' = -3$

10. $y = x^2 \cdot \ln(2+e^{-6x})$

$y' = x^2 \cdot \frac{1}{2+e^{-6x}} \cdot e^{-6x}(-6) + \ln(2+e^{-6x}) \cdot (2x)$

$= \boxed{\frac{-6x^2}{(2+e^{-6x})e^{6x}} + 2x\ln(2+e^{-6x})}$

11. $y = \ln\sqrt{x}$

$y' = \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2x}$

OR// $y = \ln\sqrt{x} = \ln(x^{1/2}) = \frac{1}{2}\ln x$

$y' = \frac{1}{2} \cdot \frac{1}{x} = \frac{1}{2x}$ Match!

12. $y = \sqrt{\ln x}$

$y' = \boxed{\frac{1}{2\sqrt{\ln x}} \cdot \frac{1}{x}}$

13. $f(x) = \ln(1+e^{2x})$

$f'(x) = \frac{1}{1+e^{2x}} \cdot e^{2x} \cdot 2$

$f'(0) = \frac{1}{1+\cancel{e^0}} \cdot \cancel{e^0} \cdot 2 = \frac{1}{\cancel{2}} \cdot \cancel{2} = \boxed{1}$

14. $f(x) = \frac{\ln x}{x}$

$$f'(x) = \frac{\cancel{x}\cdot\frac{1}{\cancel{x}} - \ln x\,(1)}{x^2} = \frac{1-\ln x}{\underset{\oplus}{x^2}} \overset{\text{set}}{=} 0$$

$$\Rightarrow 1-\ln x = 0$$

$\ln x = 1 \Rightarrow x = e$ critical number

Sign Testing into $f'(x)$

	0	1	e	10
$f'(x)$		$+$		$-$
$f(x)$		$\nearrow$	Abs. MAX	$\searrow$

Absolute Max Value $f(e) = \frac{\cancel{\ln e}^{\,1}}{e} = \frac{1}{e}$

15. $f(x) = \frac{x}{2} - \ln x$

$$f'(x) = \frac{1}{2} - \frac{1}{x} \overset{\text{set}}{=} 0$$

$\frac{1}{2} = \frac{1}{x} \Rightarrow x = 2$ critical number

SignTesting into $f'(x)$

	0	1	2	10
$f'(x)$		$-$		$+$
$f(x)$		$\searrow$	Abs MIN	$\nearrow$

Absolute Minimum Value $f(2) = \frac{2}{2} - \ln 2 = 1 - \ln 2$

16. $\cancel{\ln e}^{3} + e^{-4\ln 2} = 3 + \cancel{e}^{\cancel{\ln}(2^{-4})} = 3 + 2^{-4} = 3 + \frac{1}{2^4} = 3 + \frac{1}{16} = \frac{48}{16} + \frac{1}{16} = \frac{49}{16}$

17. $\cancel{e}^{\cancel{\ln}(\ln 3)} + \frac{1}{3}\ln 8 = \ln 3 + \ln(8^{1/3}) = \ln 3 + \ln \cancel{\sqrt[3]{8}}^{\,2}$

$= \ln 3 + \ln 2 = \ln(3\cdot 2) = \ln 6$

18. $\ln\frac{1}{\sqrt{e}} + \sqrt{e^{2\ln 3}} = \cancel{\ln}(\cancel{e}^{-1/2}) + \sqrt{\cancel{e}^{\cancel{\ln}(3^2)}} = -\frac{1}{2} + \cancel{\sqrt{9}}^{\,3} = -\frac{1}{2} + \frac{6}{2} = \frac{5}{2}$

19. $2\ln x - 3\ln y - 4\ln z = \ln(x^2) - \ln(y^3) - \ln(z^4)$

$$= \ln\left(\frac{x^2}{y^3}\right) - \ln(z^4)$$

$$= \ln\left(\frac{\frac{x^2}{y^3}}{\frac{z^4}{1}}\right)^{\frac{1}{z^4}} = \boxed{\ln\left(\frac{x^2}{y^3 z^4}\right)}$$ Match!

OR//

$2\ln x - 3\ln y - 4\ln z = \ln(x^2) - \ln(y^3) - \ln(z^4)$

$$= \ln(x^2) - \left(\ln(y^3) + \ln(z^4)\right)$$

$$= \ln(x^2) - \left(\ln(y^3 \cdot z^4)\right)$$

$$= \ln\left(\frac{x^2}{y^3 \cdot z^4}\right)$$ Match!

20. $\int_{-e^2}^{-e} \frac{5}{x}\,dx = 5\int_{-e^2}^{-e} \frac{1}{x}\,dx = 5\ln|x| \Big|_{-e^2}^{-e} = 5\left(\ln|\cancel{-}e| - 5\ln|\cancel{-}e^2|\right)$

$$= 5\left(\cancel{\ln|e|}^{1} - \ln|\cancel{e^2}|^{2}\right) = 5(1-2) = -5$$

21. $\int_0^3 \frac{1}{5x+1}\,dx = \frac{1}{5}\int_1^{16} \frac{1}{u}\,du = \frac{1}{5}\ln|u| \Big|_1^{16} = \frac{1}{5}\left(\ln|16| - \cancel{\ln|1|}^{0}\right)$

$$= \frac{\ln(16)}{5}$$

$u = 5x+1$
$du = 5dx$
$\frac{1}{5}du = dx$

$x=0 \Rightarrow u = 5(0)+1 = 1$
$x=3 \Rightarrow u = 5(3)+1 = 16$

22. $\int_e^{e^5} \frac{1}{x\ln x}\,dx = \int_1^5 \frac{1}{u}\,du = \ln|u| \Big|_1^5 = \ln|5| - \cancel{\ln|1|}^{0}$

$$= \ln 5$$

$u = \ln x$
$du = \frac{1}{x}dx$

$x = e \Rightarrow u = \cancel{\ln e} = 1$
$x = e^5 \Rightarrow u = \cancel{\ln e^5} = 5$

23. $\displaystyle\int \frac{x^6}{2-x^7}\,dx = -\frac{1}{7}\int \frac{1}{u}\,du = -\frac{1}{7}\ln|u| + C$

$= -\frac{1}{7}\ln\left|2-x^7\right| + C$

$$\boxed{\begin{aligned} u &= 2-x^7 \\ du &= -7x^6\,dx \\ -\tfrac{1}{7}\,du &= x^6\,dx \end{aligned}}$$

24. $\displaystyle\int_0^{\ln 2} \frac{e^{3x}}{8+e^{3x}}\,dx = \frac{1}{3}\int_9^{16} \frac{1}{u}\,du = \frac{1}{3}\ln|u|\Big|_9^{16} = \frac{1}{3}\left(\ln|16| - \ln|9|\right)$

$= \frac{1}{3}\ln\left(\frac{16}{9}\right)$

$$\boxed{\begin{aligned} u &= 8+e^{3x} \\ du &= 3e^{3x}\,dx \\ \tfrac{1}{3}\,du &= e^{3x}\,dx \end{aligned}}$$

$$\boxed{\begin{aligned} x = 0 &\Rightarrow u = 8+e^0 = 9 \\ x = \ln 2 &\Rightarrow u = 8+e^{3\ln 2} \\ &= 8+\cancel{e}^{\cancel{\ln}(2^3)} \\ &= 8+8 = 16 \end{aligned}}$$

25. $\displaystyle\int_1^2 \frac{1+x^3}{x^4}\,dx =$ split $\displaystyle\int_1^2 \frac{1}{x^4} + \frac{x^3}{x^4}\,dx = \int_1^2 x^{-4} + \frac{1}{x}\,dx$

Algebra

$= \frac{x^{-3}}{-3} + \ln|x|\Big|_1^2 = -\frac{1}{3x^3} + \ln|x|\Big|_1^2$

$= -\frac{1}{3\cdot 8} + \ln 2 - \left(-\frac{1}{3} + \cancel{\ln 1}^{\,0}\right)$

$= -\frac{1}{24} + \ln 2 + \frac{1}{3} = -\frac{1}{24} + \ln 2 + \frac{8}{24} = \ln 2 + \frac{7}{24}$