

Homework 16 Answer Key

Also WS #10

Spring 22

1. $y = x \ln x - x$

$$y' = \cancel{x} \cdot \frac{1}{\cancel{x}} + \ln x (1) - 1 = 1 + \ln x - 1 = \ln x$$

2. $y = \sin(\ln x)$

$$y' = \cos(\ln x) \cdot \frac{1}{x} = \frac{\cos(\ln x)}{x}$$

3. $y = \ln(\sin x)$

$$y' = \frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x}$$

4. $f(x) = \ln\left(\frac{1}{x}\right)$

$$f'(x) = \frac{1}{\frac{1}{x}} \cdot \left(-\frac{1}{x^2}\right) = x \left(-\frac{1}{x^2}\right) = -\frac{1}{x}$$

OR $f(x) = \ln\left(\frac{1}{x}\right) = \ln(x^{-1}) = -\ln x$

$$f'(x) = -\frac{1}{x} \quad \text{Match!}$$

Log Power Rule

$$\ln(a^b) = b \ln a$$

5. $f(x) = \frac{1}{\ln x} = (\ln x)^{-1}$

$$f'(x) = -(\ln x)^{-2} \cdot \frac{1}{x} = -\frac{1}{x(\ln x)^2}$$

6. $y = \ln(\ln(\ln x))$

$$y' = \boxed{\frac{1}{\ln(\ln x)} \cdot \frac{1}{\ln x} \cdot \frac{1}{x}}$$

7. $y = (\ln x)^3$

$$y' = 3(\ln x)^2 \cdot \frac{1}{x}$$

8. $y = \ln(x^3)$

$$y' = \frac{1}{x^3} \cdot (3x^2) = \frac{3}{x}$$

OR $y = \ln(x^3) = 3 \ln x$

$$y' = 3 \cdot \frac{1}{x} = \frac{3}{x}$$

$$9. \quad y = \ln(e^{-3x})$$

$$y' = \frac{1}{e^{-3x}} \cdot e^{-3x} \cdot (-3) = -3$$

$$\text{OR} // \quad y = \ln(e^{-3x}) = -3x$$

$$y' = -3$$

$$10. \quad y = x^2 \cdot \ln(2 + e^{-6x})$$

$$y' = x^2 \cdot \frac{1}{2 + e^{-6x}} \cdot e^{-6x} (-6) + \ln(2 + e^{-6x}) \cdot (2x)$$

$$= \boxed{\frac{-6x^2}{(2 + e^{-6x})e^{6x}} + 2x \ln(2 + e^{-6x})}$$

$$11. \quad y = \ln\sqrt{x}$$

$$y' = \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2x}$$

$$\text{OR} // \quad y = \ln\sqrt{x} = \ln(x^{1/2}) = \frac{1}{2} \ln x$$

$$y' = \frac{1}{2} \cdot \frac{1}{x} = \frac{1}{2x} \quad \text{match!}$$

$$12. \quad y = \sqrt{\ln x}$$

$$y' = \boxed{\frac{1}{2\sqrt{\ln x}} \cdot \frac{1}{x}}$$

$$13. \quad f(x) = \ln(1 + e^{2x})$$

$$f'(x) = \frac{1}{1 + e^{2x}} \cdot e^{2x} \cdot 2$$

$$f'(0) = \frac{1}{1 + e^0} \cdot e^0 \cdot 2 = \frac{1}{2} \cdot 2 = \boxed{1}$$

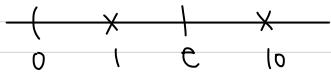
$$14. f(x) = \frac{\ln x}{x}$$

$$f'(x) = \frac{x/\frac{1}{x} - \ln x(1)}{x^2} = \frac{1 - \ln x}{x^2} \stackrel{\text{set}}{=} 0$$

$$\Rightarrow 1 - \ln x = 0$$

$$\ln x = 1 \Rightarrow x = e \quad \text{critical number}$$

Sign Testing into $f'(x)$



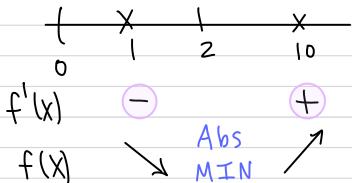
$$\text{Absolute Max Value } f(e) = \frac{\ln e}{e} = \frac{1}{e}$$

$$15. f(x) = \frac{x}{2} - \ln x$$

$$f'(x) = \frac{1}{2} - \frac{1}{x} \stackrel{\text{set}}{=} 0$$

$$\frac{1}{2} = \frac{1}{x} \Rightarrow x = 2 \quad \text{critical number}$$

Sign Testing into $f'(x)$



$$\text{Absolute Minimum Value } f(2) = \frac{2}{2} - \ln 2 = 1 - \ln 2$$

$$16. \cancel{\ln e^3 + e^{-4 \ln 2}} = 3 + \cancel{e^{\ln(2^{-4})}} = 3 + 2^{-4} = 3 + \frac{1}{2^4} = 3 + \frac{1}{16} = \frac{48}{16} + \frac{1}{16} = \frac{49}{16}$$

$$17. \cancel{e^{\ln(\ln 3)}} + \frac{1}{3} \ln 8 = \ln 3 + \ln(8^{\frac{1}{3}}) = \ln 3 + \ln \cancel{8^{\frac{1}{3}}}^2$$

$$= \ln 3 + \ln 2 = \ln(3 \cdot 2) = \ln 6$$

$$18. \ln \frac{1}{\sqrt{e}} + \sqrt{e^{2 \ln 3}} = \ln(e^{-1/2}) + \sqrt{e^{\ln(3^2)}} = -\frac{1}{2} + \sqrt{9} = -\frac{1}{2} + \frac{6}{2} = \frac{5}{2}$$

$$19. \cancel{2\ln x} - \cancel{3\ln y} - \cancel{4\ln z} = \ln(x^2) - \ln(y^3) - \ln(z^4)$$

$$= \ln\left(\frac{x^2}{y^3}\right) - \ln(z^4)$$

$$= \ln\left(\frac{\cancel{x^2}}{\cancel{y^3}}\right)^{\frac{1}{z^4}} = \boxed{\ln\left(\frac{x^2}{y^3 z^4}\right)} \text{ Match!}$$

OR

$$\cancel{2\ln x} - \cancel{3\ln y} - \cancel{4\ln z} = \ln(x^2) - \ln(y^3) - \ln(z^4)$$

$$= \ln(x^2) - (\ln(y^3) + \ln(z^4))$$

$$= \ln(x^2) - (\ln(y^3 \cdot z^4))$$

$$= \ln\left(\frac{x^2}{y^3 \cdot z^4}\right) \text{ Match!}$$

$$20. \int_{-e^2}^{-e} \frac{5}{x} dx = 5 \int_{-e^2}^{-e} \frac{1}{x} dx = 5 \ln|x| \Big|_{-e^2}^{-e} = 5 (\ln|-e| - 5\ln|-e^2|)$$

$$= 5 (\ln|e| - \ln|e^2|) = 5(1-2) = \boxed{-5}$$

$$21. \int_0^3 \frac{1}{5x+1} dx = \frac{1}{5} \int_1^{16} \frac{1}{u} du = \frac{1}{5} \ln|u| \Big|_1^{16} = \frac{1}{5} (\ln|16| - \ln|1|)$$

$$= \boxed{\frac{\ln(16)}{5}}$$

$$u = 5x+1$$

$$du = 5dx$$

$$\frac{1}{5} du = dx$$

$$x=0 \Rightarrow u=5(0)+1=1$$

$$x=3 \Rightarrow u=5(3)+1=16$$

$$22. \int_e^5 \frac{1}{x \ln x} dx = \int_1^5 \frac{1}{u} du = \ln|u| \Big|_1^5 = \ln|5| - \ln|1|$$

$$= \boxed{\ln 5}$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$x=e \Rightarrow u=\ln e=1$$

$$x=e^5 \Rightarrow u=\ln e^5=5$$

$$23. \int \frac{x^6}{2-x^7} dx = -\frac{1}{7} \int \frac{1}{u} du = -\frac{1}{7} \ln|u| + C$$

$$= -\frac{1}{7} \ln|2-x^7| + C$$

$$u = 2-x^7$$

$$du = -7x^6 dx$$

$$-\frac{1}{7} du = x^6 dx$$

$$24. \int_0^{\ln 2} \frac{e^{3x}}{8+e^{3x}} dx = \frac{1}{3} \int_9^{16} \frac{1}{u} du = \frac{1}{3} \ln|u| \Big|_9^{16} = \frac{1}{3} (\ln 16 - \ln 9)$$

$$u = 8+e^{3x}$$

$$du = 3e^{3x} dx$$

$$\frac{1}{3} du = e^{3x} dx$$

$$x=0 \Rightarrow u=8+e^0=9$$

$$x=\ln 2 \Rightarrow u=8+e^{\ln 2} = 8+e^{\ln(2^3)} = 8+8=16$$

$$= \frac{1}{3} \ln\left(\frac{16}{9}\right)$$

$$25. \int_1^2 \frac{1+x^3}{x^4} dx \stackrel{\text{split}}{=} \int_1^2 \frac{1}{x^4} + \frac{x^3}{x^4} dx = \int_1^2 x^{-4} + \frac{1}{x} dx$$

$$\text{Algebra} = \frac{x^{-3}}{-3} + \ln|x| \Big|_1^2 = -\frac{1}{3x^3} + \ln|x| \Big|_1^2$$

$$= -\frac{1}{3 \cdot 8} + \ln 2 - \left(-\frac{1}{3} + \ln 1 \right)^0$$

$$= -\frac{1}{24} + \ln 2 + \frac{1}{3} = -\frac{1}{24} + \ln 2 + \frac{8}{24} = \ln 2 + \frac{7}{24}$$