Trigonometry Review

The following sheets contain an overview of some necessary trigonometric identities, definitions, and graphs. You are expected to know the information presented here. Hopefully you have already seen these ideas before and understand them. They will be extremely useful tools, especially for differentiation and integration.

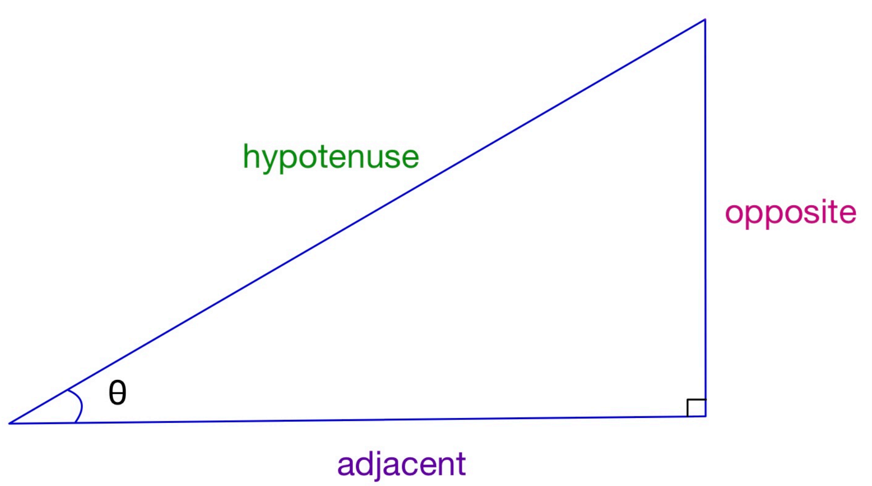
Right Triangles

Assuming the preliminaries of angle measurement in radians, we are able to define the circular functions and examine their relationships.

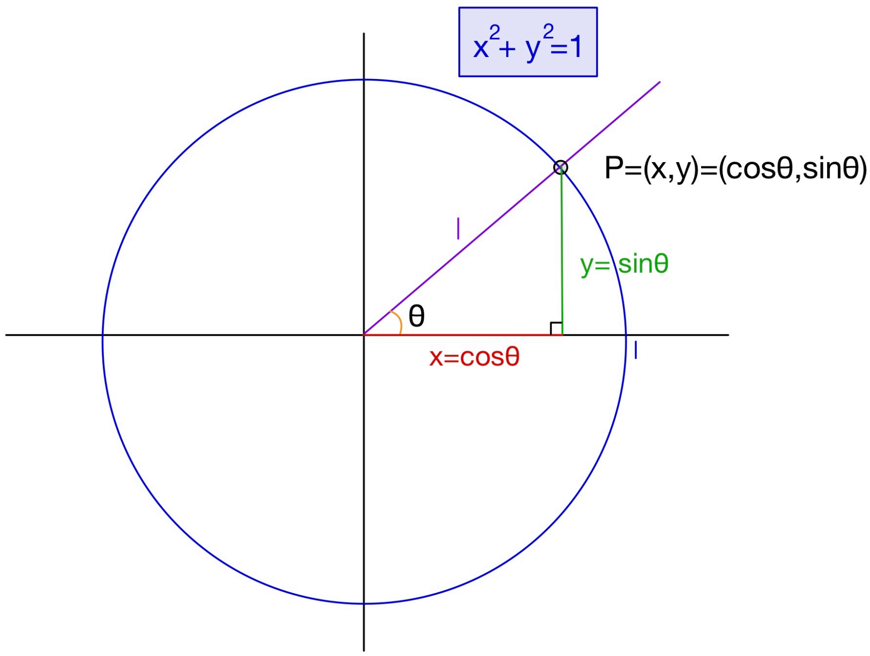
Suppose θ is a given fixed acute angle (between $0°$ and $90°$). We define the following functions with the help of our right triangle. (Remember **SOH CAH TOA**?)

$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \cos\theta = \frac{adjacent}{hypotenuse} \qquad \tan\theta = \frac{opposite}{adjacent} = \frac{\sin\theta}{\cos\theta}$$

$$\csc\theta = \frac{hyp}{opp} = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{hyp}{adj} = \frac{1}{cos\theta} \qquad \cot\theta = \frac{adj}{opp} = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$



The Unit Circle



In the figure above, a circle is drawn on the coordinate plane, with center at the origin $(0,0)$ and radius equal to 1. The equation of the graph is $x^2+y^2=1$. Again suppose θ is a fixed angle, and consider the right triangle formed, as in the figure. Since the hypotenuse has length 1 here (remember it's a *unit* circle), using the previous formulas we see that

$$\sin\theta = \frac{opp}{1} = opp \qquad \text{and} \qquad \cos\theta = \frac{adj}{1} = adj$$

respectively. Therefore,

$$\cos\theta = x \text{ coordinate of the point } P$$

and

$$\sin\theta = y \text{ coordinate of the point } P.$$

This means any point P on the unit circle may be represented as

$$(\cos\theta, \sin\theta)$$

where θ is the angle formed by the line (through the origin and P) and the x-axis. There is an advantage to using the unit circle representation. It allows us to find trig. values for all angles; we are not restricted to acute angles, as with the right triangle representation.

Identities

Below is a summary of some nice properties of trigonometric functions. You should *definitely* know the first two!! There are others not listed here which you may find in your book. Remember the Pythagorean Theorem? That will help you understand the first identity.

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x$$

$$\sin(-x) = -\sin x$$
$$\cos(-x) = \cos x$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$
$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\sin(2x) = 2\sin x \cos x$$
$$\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

Angle Measurement

Angles may be measured in either degrees or radians. The conversion is given here:

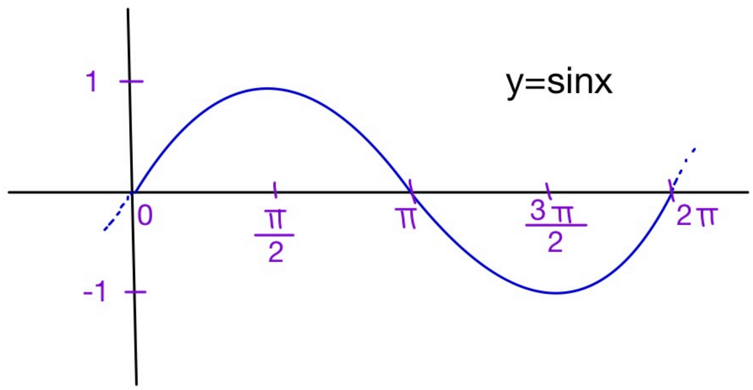
$$\pi \text{ rad} = 180^\circ$$

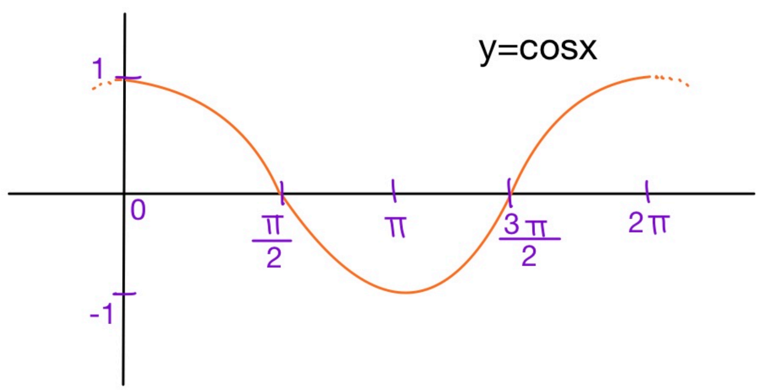
which implies

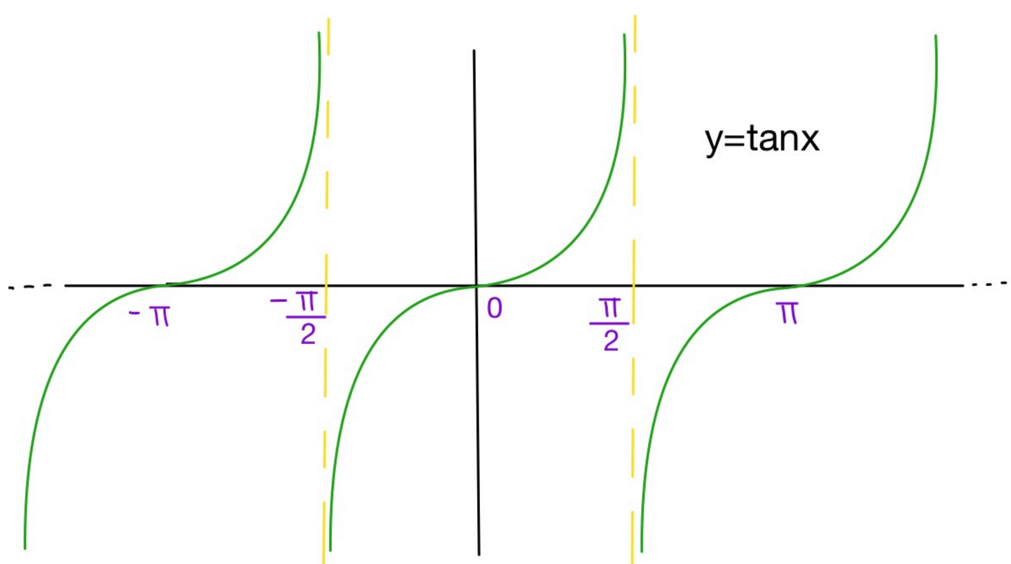
$$1^\circ = \frac{\pi}{180} \text{ rad} \qquad \text{or} \qquad 1 \text{ rad} = \frac{180^\circ}{\pi}.$$

Graphs

It is helpful to visualize the graphs of some of the trigonometric functions. The partial graphs of the sine, cosine, and tangent functions are contained here. You will find others in your text book. Get to know these three plots **very** well!







Special Angles

The chart below will assist you in remembering certain values of trigonometric functions. However, you should be able to calculate these without looking at the sheet. Use the unit circle, right triangles, or the wave plots of the functions as reference. Draw these if you have to, and get used to it until you can easily visualize the circle and angles.

Angle(radians)	Angle(degrees)	Sine	Cosine	Tangent
0	0	0	1	0
$\pi/6$	$30°$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
$\pi/4$	$45°$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\pi/3$	$60°$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$\pi/2$	$90°$	1	0	$--$
$2\pi/3$	$120°$	$\sqrt{3}/2$	$-1/2$	$-\sqrt{3}$
$3\pi/4$	$135°$	$\sqrt{2}/2$	$-\sqrt{2}/2$	-1
$5\pi/6$	$150°$	$1/2$	$-\sqrt{3}/2$	$-1/\sqrt{3}$
π	$180°$	0	-1	0
$7\pi/6$	$210°$	$-1/2$	$-\sqrt{3}/2$	$1/\sqrt{3}$
$5\pi/4$	$225°$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	1
$4\pi/3$	$240°$	$-\sqrt{3}/2$	$-1/2$	$\sqrt{3}$
$3\pi/2$	$270°$	-1	0	$--$
$5\pi/3$	$300°$	$-\sqrt{3}/2$	$1/2$	$-\sqrt{3}$
$7\pi/4$	$315°$	$-\sqrt{2}/2$	$\sqrt{2}/2$	-1
$11\pi/6$	$330°$	$-1/2$	$\sqrt{3}/2$	$-1/\sqrt{3}$
2π	$360°$	0	1	0

Try filling in the following chart. These are the values you will be expected to know. Practice them.

Angle(radians)	Angle(degrees)	Sine	Cosine	Tangent
0	0			
$\pi/6$	$30°$			
$\pi/4$	$45°$			
$\pi/3$	$60°$			
$\pi/2$	$90°$			
π	$180°$			
$3\pi/2$	$270°$			
2π	$360°$			

You should find the following triangles helpful.