Math 106, Benedetto

What you need to know for Exam 2

This Exam covers Chapter 4 on Integration, including the Limit Definition using Riemann Sums, and then the Fundamental Theorem of Calculus (both Parts). We studied how to make Estimate using Approximating Rectangles, and then we derived the Limit formula to compute the actual *net* or *signed* area of a bounded region. We also studied how to manually use Area Interpretations to check the Area computation for some simpler functions. Our integration work included lighter computations, as well as how to use Algebra to antidifferentiate complex Products and Quotients. Finally, we used cases to antidifferentiate functions involving Absolute Values. We also covered the computations and formulas for Displacement and Total Distance. Along the way, we still need to work with antiderivatives and values for Trig Functions. The following is a list of most of the topics covered. **THIS IS NOT A COMPREHENSIVE LIST, BUT MERELY AN AID.** Remember, no calculators in any exams.

- 4.1: Areas and Distances. Know the idea of the Riemann sum as a way to approximate area or distance. We mostly focussed on the right endpoint case using n equal-width intervals. Review Sigma notation.
- 4.2: The Definite Integral. The definition of the definite integral as a limit of Riemann sums of Heights times Widths. Notation for the definite integral. Evaluating definite integrals using Sigma identities and limit laws. Know the Properties of the definite integral.
- 4.3: FTC. The Fundamental Theorem of Calculus, Parts I and II. Know the statements of both parts (they're stated together in the book), and know how to use them to differentiate integral-defined functions and to compute definite integrals.
- 4.4: Indefinite Integrals. Know the definition of indefinite integrals, and make sure not to confuse them with definite integrals. Know the table of standard, elementary antiderivatives. Be able to evaluate some integrals, sometimes using algebra. Know how to use velocity to compute both displacement and total distance traveled. Remember $+C$ for all indefinite integrals.
- 4.5: Integration using u-Substitution. Know the substitution rule for indefinite integrals and for definite integrals. Practice **a lot** to make sure you can use them. Make sure to **change** (or mark) your limits of integration in u-substitution.

Techniques of Integration

1. "We Know It" Snap facts
2. Algebra
3. u-substitution

Common Types of Problems

- Know how to compute an integral from the Limit Definition (i.e., chop up the interval, make the Riemann sum, use Sigma identities, and take the limit). Write all formulas on your cheat sheet.

- Know how to compute a definite integral in multiple ways: 1. using the Limit Definition, 2. using FTC, and maybe 3. Area Interpretations in some situations.

- Practice computing integrals, both definite and indefinite. Remember, you have two main tools: you can simplify the integrand using algebra, and/or you can do a substitution. Those are basically your only choices in Calculus I. The idea is to reduce the integrand to something you actually know the antiderivative of. The key thing is to **keep trying different ideas until something works**. Practice a combined technique of speed and accuracy!!!

  TIP: Remember if you are given a function involving a Product or Quotient of powers of x then you must simplify using algebra and break it down into simpler pieces and finish using Power Rules. Always **prep** your powers to the numerator using exponential algebra. Review all exponential algebra for simplifying your powers. When it's time to evaluate using the limits of integration, it's advised to move the negative power back into the denominator using exponential algebra rules, *before* evaluating at the two endpoints.

- Solve Physics type word problems, involving linear motion. Compute Position, Velocity or Acceleration given any of the other motion equations. Remember which direction is the Derivative and which is the Antiderivative. A problem may give you Velocity and ask for Position and Acceleration, so you might use *both* differentiation and antidifferentiation. Write down the given info as that might help you solve for $+C$.

- Given velocity, be able to compute Displacement and Total Distance. Of course, Total Distance is usually harder to muscle because of Absolute Value, and the need for split cases. Drawing the graph may help figure out the cases.

- Solve Initial Valued Differential Equations for a specific Antiderivative. For this Exam, the antidifferentiation may involve u-substitution. Use the given function value to solve for $+C$.

- Compute derivatives of *variable limit* integrals using the Fundamental Theorem of Calculus Part I.

- Compute integrals involving u-substitution for **both** Indefinite (which gains $+C$) and Definite Integrals (which requires a change of the limits using the choice of u).