Math 106 Practice Exam #2 Spring 2014

1(a) $\displaystyle\int_{-1}^{2} 5+x-x^2\,dx = 5x+\frac{x^2}{2}-\frac{x^3}{3}\Bigg|_{-1}^{2} = 10+2-8/3-\left(-5+\frac{1}{2}+\frac{1}{3}\right)$

$= 12-8/3+5-\frac{1}{2}-\frac{1}{3}$

$= 17-\frac{1}{2}-\frac{9}{3}$

$= 17-1/2-3$

$= 14-1/2 = \boxed{\frac{27}{2}}$

(b) $f(x)=5+x-x^2 \quad a=-1,\ b=2,\ \Delta x = \frac{b-a}{n} = \frac{2-(-1)}{n} = \frac{3}{n},\ x_i = a+i\Delta x = -1+\frac{3i}{n}$

$$\int_{-1}^{2} 5+x-x^2\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty}\sum_{i=1}^{n} f\left(-1+\frac{3i}{n}\right)\left[\frac{3}{n}\right]$$

$$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n} 5+\left(-1+\frac{3i}{n}\right)-\left(-1+\frac{3i}{n}\right)^2$$

$$-\left(1-6i/n+\frac{9i^2}{n^2}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n} 5-1+\frac{3i}{n}-1+\frac{6i}{n}-\frac{9i^2}{n^2}$$

$$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n} 3+\frac{9i}{n}-\frac{9i^2}{n^2}$$

$$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n}3+\frac{3}{n}\sum_{i=1}^{n}\frac{9i}{n}-\frac{3}{n}\sum_{i=1}^{n}\frac{9i^2}{n^2}$$

$$= \lim_{n\to\infty}\frac{9}{\cancel{n}}\cancelto{n}{\sum_{i=1}^{n}1}+\frac{27}{n^2}\sum_{i=1}^{n}i-\frac{27}{n^3}\sum_{i=1}^{n}i^2$$

$$= \lim_{n\to\infty} 9+\frac{27}{n^2}\left[\frac{n(n+1)}{2}\right]-\frac{27}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty} 9+\frac{27}{2}\cancelto{1}{\left(\frac{n}{n}\right)}\cancelto{1}{\Bigg(\underbrace{\frac{n+1}{n}}_{1+1/n}\Bigg)}-\frac{27}{6}\cancelto{1}{\left(\frac{n}{n}\right)}\cancelto{1}{\Bigg(\underbrace{\frac{n+1}{n}}_{1+1/n}\Bigg)}\cancelto{\boxed{2}}{\Bigg(\underbrace{\frac{2n+1}{n}}_{2+1/n}\Bigg)}$$

$$= 9+\frac{27}{2}-\frac{54}{6} = \cancel{9}+\frac{27}{2}-\cancel{9} = \boxed{\frac{27}{2}}$$

2. $\frac{d}{dx}\int_x^7 \frac{1-\sin t}{t^2+\tan t+9}\,dt = \frac{d}{dx}\left[-\int_7^x \frac{1-\sin t}{t^2+\tan t+9}\,dt\right] = \boxed{\frac{-(1-\sin x)}{x^2+\tan x+9}}$

3. $\int \frac{(x+1)(x+2)}{\sqrt{x}}\,dx = \int \frac{x^2+3x+2}{\sqrt{x}}\,dx = \int x^{3/2}+3x^{1/2}+2x^{-1/2}\,dx$

$= \frac{x^{5/2}}{(5/2)} + \frac{3x^{3/2}}{(3/2)} + \frac{2x^{1/2}}{1/2} + C = \boxed{\frac{2}{5}x^{5/2} + 2x^{3/2} + 4x^{1/2} + C}$

4. $\int \frac{y^3-9y\sin y+26y^{-1}}{y}\,dy = \int y^2-9\sin y+26y^{-2}\,dy$

$= y^3/3 + 9\cos y + \frac{26y^{-1}}{-1} + C$

$= \boxed{y^3/3 + 9\cos y - 26/y + C}$

5. $\int \sqrt{x}\cos(x\sqrt{x})\,dx = \int \sqrt{x}\cos(x^{3/2})\,dx = \frac{2}{3}\int \cos u\,du = \frac{2}{3}\sin u + C$

$= \boxed{\frac{2}{3}\sin(x^{3/2}) + C}$

$$\boxed{\begin{array}{l} u = x^{3/2} \\ du = \frac{3}{2}x^{1/2}dx \\ \frac{2}{3}du = x^{1/2}dx \end{array}}$$

6. $\int_{-2}^{2} |1-x|\,dx = \underbrace{\int_{-2}^{1}|1-x|\,dx}_{x<1} + \underbrace{\int_{1}^{2}|1-x|\,dx}_{x\geq 1} = \int_{-2}^{1} 1-x\,dx + \int_1^2 \overset{-(1-x)}{x-1}\,dx$

Recall $|1-x| = \begin{cases} 1-x & \text{if } 1-x \geq 0 \;\to x\leq 1 \\ -(1-x) & \text{if } 1-x<0 \;\to x>1 \end{cases}$

$= x - \frac{x^2}{2}\Big|_{-2}^{1} + \frac{x^2}{2} - x\Big|_1^2$

$= 1 - \frac{1}{2} - (-2-2) + \cancel{2-2} - \left(\frac{1}{2} - 1\right)$

$= \frac{1}{2} + 4 + \frac{1}{2} = \boxed{5}$

Repeat from HW #14

7. $\int 7\cos(5x) - 5\sin(7x)\,dx = 7\int \cos(5x)dx - 5\int \sin(7x)dx$

$\boxed{\begin{array}{c} u=5x \\ du=5dx \\ \frac{1}{5}du=dx \end{array}}$ $= \frac{7}{5}\int \cos u\,du - \frac{5}{7}\int \sin w\,dw$ $\boxed{\begin{array}{c} w=7x \\ dw=7dx \\ \frac{1}{7}dw=dx \end{array}}$

$= \frac{7}{5}\sin u + \frac{5}{7}\cos w$

$= \boxed{\frac{7}{5}\sin(5x) + \frac{5}{7}\cos(7x) + C}$

Repeat from HW#14

8. $\int (x^{-7/2} + x^{-1/3})\underbrace{\sqrt{x}}_{x^{1/2}}\,dx = \int x^4 + x^{\frac{-2}{6}+\frac{3}{6}}\,dx = \int x^4 + x^{1/6}\,dx$

$= \boxed{\frac{x^5}{5} + \frac{6}{7}x^{7/6} + C}$

9. $\int_{-1}^{3} \frac{1}{(x+2)^2}\,dx = \int_1^5 \frac{1}{u^2}\,du = \int_1^5 u^{-2}\,du = \frac{u^{-1}}{-1}\Big|_1^5 = \frac{-1}{u}\Big|_1^5 = -\frac{1}{5} - (-1) = \boxed{\frac{4}{5}}$

$\frac{5}{5}$

$\boxed{\begin{array}{c} u=x+2 \\ du=dx \end{array}}$ $\boxed{\begin{array}{c} x=-1 \Rightarrow u=1 \\ x=3 \Rightarrow u=5 \end{array}}$

10. $\int_{\pi^2/9}^{\pi^2/4} \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx = 2\int_{\pi/3}^{\pi/2} \cos u\,du = 2\sin u\Big|_{\pi/3}^{\pi/2} = 2\left[\sin\frac{\pi}{2} - \sin\frac{\pi}{3}\right]$ (1, $\sqrt{3}/2$)

$= 2\left[1 - \frac{\sqrt{3}}{2}\right]$

$= \cancel{2}\left[\frac{2-\sqrt{3}}{\cancel{2}}\right] = \boxed{2-\sqrt{3}}$

$\boxed{\begin{array}{c} u=\sqrt{x} \\ du=\frac{1}{2\sqrt{x}}dx \\ 2du=\frac{1}{\sqrt{x}}dx \end{array}}$ $\boxed{\begin{array}{c} x=\pi^2/9 \Rightarrow u=\pi/3 \\ x=\pi^2/4 \Rightarrow u=\pi/2 \end{array}}$

11. $\int_1^4 \frac{x^4-8}{x^2}\,dx = \int_1^4 x^2 - 8x^{-2}\,dx = \frac{x^3}{3} - \frac{8x^{-1}}{(-1)}\Big|_1^4 = \frac{x^3}{3} + \frac{8}{x}\Big|_1^4$

$= \frac{64}{3} + 2 - \left(\frac{1}{3} + 8\right) = \frac{63}{3} - 6 = 21 - 6 = \boxed{15}$

$-1/3 - 8$

12. $$\int_0^4 \frac{1}{\sqrt{2x+1}}\,dx = \frac{1}{2}\int_1^9 \frac{1}{\sqrt{u}}\,du = \frac{1}{2}\int_1^9 u^{-1/2}\,du = \cancel{\frac{1}{2}}\,\frac{u^{1/2}}{\cancel{1/2}}\Bigg|_1^9$$
$$= \sqrt{9} - \sqrt{1}$$
$$= 3 - 1 = \boxed{2}$$

$$\boxed{\begin{aligned} u &= 2x+1 \\ du &= 2\,dx \\ \tfrac{1}{2}\,du &= dx \end{aligned}} \qquad \boxed{\begin{aligned} x=0 &\Rightarrow u=1 \\ x=4 &\Rightarrow u=9 \end{aligned}}$$

13. $$\int_{-1}^{1} \frac{x}{(1+x^2)^4}\,dx = \frac{1}{2}\int_2^2 \frac{1}{u^4}\,du = \boxed{0}$$

Note: original integrand is an odd function [f(-x)= f(x
so by symmetry equals
0 . Makes Sense.

$$\boxed{\begin{aligned} u &= 1+x^2 \\ du &= 2x\,dx \\ \tfrac{1}{2}\,du &= x\,dx \end{aligned}} \qquad \boxed{\begin{aligned} x=-1 &\Rightarrow u=2 \\ x=1 &\Rightarrow u=2 \end{aligned}}$$

14. $$\int_{-2}^{-1} \left(x - \frac{5}{x^3}\right)^2 dx = \int_{-2}^{-1} x^2 - \frac{10}{x^2} + \frac{25}{x^6}\,dx = \int_{-2}^{-1} x^2 - 10x^{-2} + 25x^{-6}\,dx$$

$$= \frac{x^3}{3} - \frac{10x^{-1}}{(-1)} + \frac{25x^{-5}}{(-5)}\Bigg|_{-2}^{-1} = \frac{x^3}{3} + \frac{10}{x} - \frac{5}{x^5}\Bigg|_{-2}^{-1}$$

$$= \frac{-1}{3} + \frac{10}{(-1)} - \frac{5}{(-1)^5} - \left(-\frac{8}{3} + \frac{10}{(-2)} - \frac{5}{(-2)^5}\right)$$

$$= -\frac{1}{3} \cancel{-10} \cancel{+5} + \frac{8}{3} \cancel{+5} - \frac{5}{32}$$

$$= \frac{7}{3} - \frac{5}{32} = \frac{224}{96} - \frac{15}{96} = \boxed{\frac{209}{96}}$$

$32 \times 7 = 224$ $\quad$ $224 - 15 = 9$

15. $$\int_0^{\sqrt{5}} x\sqrt{9-x^2}\,dx = -\frac{1}{2}\int_9^4 \sqrt{u}\,du = -\frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{3}\right)u^{3/2}\Bigg|_9^4 = \frac{-1}{3}\left[(4)^{3/2} - 9^{3/2}\right] = \frac{-1}{3}\left[8 - 27\right]$$
$$= \boxed{\frac{19}{3}}$$

$$\boxed{\begin{aligned} u &= 9-x^2 \\ du &= -2x\,dx \\ -\tfrac{1}{2}\,du &= x\,dx \end{aligned}} \qquad \boxed{\begin{aligned} x=0 &\Rightarrow u=9 \\ x=\sqrt{5} &\Rightarrow u=4 \end{aligned}}$$

16. $\displaystyle\int_0^{\frac{\sqrt{\pi}}{2}} x\sin^3(x^2)\cos(x^2)\,dx = \frac{1}{2}\int_0^{\frac{1}{\sqrt{2}}} u^3\,du = \frac{1}{2}\left(\frac{u^4}{4}\right)\Bigg|_0^{1/\sqrt{2}}$

$$\boxed{\begin{aligned} u&=\sin(x^2)\\ du&=\cos(x^2)(2x)\,dx\\ \tfrac{1}{2}du&=\cos(x^2)\,x\,dx\end{aligned}} \qquad \boxed{\begin{aligned} &x=0 \Rightarrow u=\sin 0=0\\ &x=\sqrt{\pi}/2 \Rightarrow u=\sin\left(\tfrac{\pi}{4}\right)=\tfrac{\sqrt{2}}{2}\end{aligned}}$$

$$\frac{1}{8}\left[\left(\frac{1}{\sqrt{2}}\right)^4-0\right]=\frac{1}{8}\left[\frac{1}{4}\right]=\boxed{\frac{1}{32}}$$

17. $\displaystyle\int_0^{\pi/4}(1+\tan x)^3\sec^2 x\,dx = \int_1^2 u^3\,du = \frac{u^4}{4}\Bigg|_1^2 = 4-\frac{1}{4} = \boxed{\frac{15}{4}}$

$$\boxed{\begin{aligned} u&=1+\tan x\\ du&=\sec^2 x\,dx\end{aligned}} \qquad \boxed{\begin{aligned} &x=0 \Rightarrow u=1+\overset{0}{\tan 0}=1\\ &x=\pi/4 \Rightarrow u=1+\underset{1}{\tan \pi/4}=2\end{aligned}}$$

18. $\displaystyle\int x\sqrt[3]{3+x^2}\,dx = \frac{1}{2}\int\sqrt[3]{u}\,du = \frac{1}{2}\int u^{1/3}\,du = \frac{1}{2}\left(\frac{3}{4}\right)u^{4/3}+C$

$$\boxed{\begin{aligned} u&=3+x^2\\ du&=2x\,dx\\ \tfrac{1}{2}du&=x\,dx\end{aligned}}$$

$$=\boxed{\frac{3}{8}(3+x^2)^{4/3}+C}$$

19. $\displaystyle\int x^3\sqrt[3]{3+x^2}\,dx = \int x^2\cdot x\sqrt[3]{3+x^2}\,dx = \frac{1}{2}\int(u-3)\sqrt[3]{u}\,du = \frac{1}{2}\int(u-3)u^{1/3}\,du$

$$\boxed{\begin{aligned} u&=3+x^2\\ du&=2x\,dx\\ \tfrac{1}{2}du&=x\,dx\end{aligned}} \Rightarrow x^2=u-3$$

$$=\frac{1}{2}\int u^{4/3}-3u^{1/3}\,du$$

$$=\frac{1}{2}\left[\frac{3}{7}u^{7/3}-3\left(\frac{3}{4}\right)u^{4/3}\right]+C$$

$$=\boxed{\frac{1}{2}\left[\frac{3}{7}(3+x^2)^{7/3}-\frac{9}{4}(3+x^2)^{4/3}\right]+C}$$

can distribute if want

20. Let $W(t)$ = Amount Water in Tank at time t

$W'(t) = 50 - t$ rate of water flow into tank

$$W(30) - W(0) = \int_0^{30} 50 - t\,dt = 50t - \frac{t^2}{2}\Big|_0^{30}$$

$$= 1500 - \frac{900}{2} - (0 - 0) = \frac{3000}{2} - \frac{900}{2} = \frac{2100}{2} = \boxed{1050}\,\ell$$

21. $v(t) = \sin t + 1 \qquad s(0) = 3$

(a) $a(t) = \boxed{\cos t}$

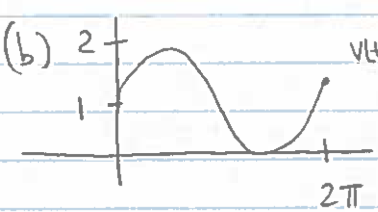
$s(t) = \int v(t)\,dt = \int \sin t + 1\,dt = -\cos t + t + C$ General Antiderivative

use condition $s(0) = 3$ to solve for C

$$s(0) = -\cancel{\cos 0}^{\,1} + 0 + C \overset{\text{set}}{=} 3 \Rightarrow C = 4$$

$\Rightarrow s(t) = \boxed{-\cos t + t + 4}$ Specific Antiderivative

(b)



$v(t)$ = Graph of $\sin t$ shifted up by 1. $v(t)$ always non-negative, so $s(t)$ is never decreasing (moving left). Velocity equals 0 at one moment but never stops for any length of time. Position function increasing $\Rightarrow$ moving to right.

(c) Total Distance $= \int_{\pi/2}^{2\pi} \cancel{|\sin t + 1|}^{\,\sin t + 1 \text{ never negative}}\,dt = \int_{\pi/2}^{2\pi} \sin t + 1\,dt = -\cos t + t \Big|_{\pi/2}^{2\pi}$

$$= -\cancel{\cos(2\pi)}^{\,1} + 2\pi - \left(-\cancel{\cos(\pi/2)}^{\,0} + \pi/2\right)$$

$$= -1 + 2\pi - \pi/2 = -1 + \frac{3\pi}{2} = \boxed{\frac{3\pi - 2}{2}} \text{ feet}$$