Math 106 Practice Exam #2 Spring 2014

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or webpages, or other aids are permitted.

Limit Definition of the Definite Integral

1. Compute $\int_{-1}^{2} 5 + x - x^2 \, dx$ using two different methods:
   (a) Fundamental Theorem of Calculus
   (b) Limit Definition of the Definite Integral.

Differentiation

2. Compute $\dfrac{d}{dx} \displaystyle\int_x^7 \frac{1 - \sin t}{t^2 + \tan t + 9} \, dt$

Integration Evaluate each of the following integrals:

3. $\displaystyle\int \frac{(x+1)(x+2)}{\sqrt{x}} \, dx$
4. $\displaystyle\int \frac{y^3 - 9y \sin y + 26y^{-1}}{y} \, dy$
5. $\displaystyle\int \sqrt{x} \cos(x\sqrt{x}) \, dx$
6. $\displaystyle\int_{-2}^{2} |1 - x| \, dx$
7. $\displaystyle\int 7\cos(5x) - 5\sin(7x) \, dx$
8. $\displaystyle\int (x^{\frac{7}{2}} + x^{-\frac{1}{3}})\sqrt{x} \, dx$
9. $\displaystyle\int_{-1}^{3} \frac{1}{(x+2)^2} \, dx$
10. $\displaystyle\int_{\frac{\pi^2}{9}}^{\frac{\pi^2}{4}} \frac{\cos\sqrt{x}}{\sqrt{x}} \, dx$
11. $\displaystyle\int_1^4 \frac{x^4 - 8}{x^2} \, dx$
12. $\displaystyle\int_0^4 \frac{1}{\sqrt{2x+1}} \, dx$
13. $\displaystyle\int_{-1}^{1} \frac{x}{(1+x^2)^4} \, dx$

14. $\displaystyle\int_{-2}^{-1} \left(x - \frac{5}{x^3}\right)^2 \, dx$

15. $\displaystyle\int_{0}^{\sqrt{5}} x\sqrt{9 - x^2} \, dx$

16. $\displaystyle\int_{0}^{\frac{\sqrt{\pi}}{2}} x \sin^3(x^2)\cos(x^2) \, dx$

17. $\displaystyle\int_{0}^{\frac{\pi}{4}} (1 + \tan x)^3 \sec^2 x \, dx$

18. $\displaystyle\int x \sqrt[3]{3 + x^2} \, dx$

19. $\displaystyle\int x^3 \sqrt[3]{3 + x^2} \, dx$ *challenge*

Displacement–Total Distance–Net Change

20. Supppose that water is pumped into an initially empty tank. The rate of water flow into the tank at time t (in seconds) is $50 - t$ liters per second. How much water flows into the tank during the first 30 seconds?

21. Consider an object moving on the number line such that its velocity at time t is

    $v(t) = \sin(t) + 1$ ft/sec. Also assume that $s(0) = 3$ ft, where as usual $s(t)$ is the position of the object at time t.

    (a) Compute the acceleration $a(t)$ and position $s(t)$.

    (b) Draw the graph of $v(t)$ for $0 \leq t \leq 2\pi$ and explain why the object is always moving to the right.

    (c) Compute the total distance travelled for $\pi/2 \leq t \leq 2\pi$.