Math 106 Midterm Exam #1 February 15, 2017

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.

• Simplify numerical answers such as $\sin\left(\frac{\pi}{6}\right)$ and $4^{\frac{3}{2}}$.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [22 Points] Differentiate each of the following functions. **Do not** simplify your answers.

(a) $f(x) = \sqrt{\sin x}$

(b) $f(x) = \sin\sqrt{x}$

(c) $f(x) = \sin^2(\tan x)$

(d) $f(x) = \sin x \cdot \tan\left(\frac{7}{x^6}\right)$

(e) $f(x) = \left(\frac{\cos(7x)}{x^2 + \sec x}\right)^{\frac{7}{8}}$

2. [20 Points]

(a) Let $f(x) = \frac{1}{\tan^2 x} + \cos^2 x + \sec(2x)$. Compute $f'\left(\frac{\pi}{6}\right)$. Simplify.

(b) Let $f(x) = 4\sin\left(x - \frac{\pi}{4}\right) - \cos x - \tan^2 x$. Show that $f'\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$.

3. [12 Points] Let $f(x) = \frac{5}{6}x + x^{\frac{5}{6}} + \frac{1}{x^{\frac{5}{6}}} + \frac{6}{5} + \frac{5}{6x^6} - \frac{6}{x^5}$.

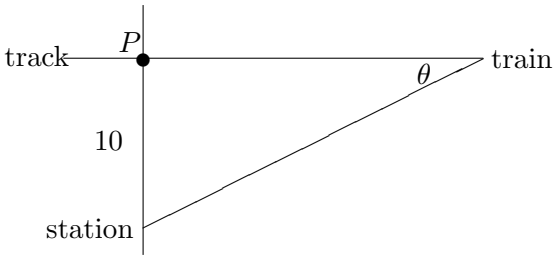
Compute the most general antiderivative

$$\int f(x)\,dx = \int \frac{5}{6}x + x^{\frac{5}{6}} + \frac{1}{x^{\frac{5}{6}}} + \frac{6}{5} + \frac{5}{6x^6} - \frac{6}{x^5}\,dx$$ Do not simplify.

4. [16 Points] Consider a point P on a train track. Suppose a train depot station is 10 feet directly south from this point P. The train is travelling east at 6 feet per second. Consider the angle as shown in the diagram. How fast is this angle changing when 2 seconds has passed since the train passed point P.

• Diagram

The picture at arbitrary time t is:



5. [20 Points]

(a) Consider a function f such that $f'(x) = \dfrac{x^{\frac{1}{5}} + x^{-\frac{2}{3}}}{x^{\frac{2}{3}}}$. Compute $f(x)$.

(b) Consider a function f such that $f''(x) = \pi \sin x + 2\cos x$ and $f'\left(\dfrac{\pi}{2}\right) = 0$ and $f(\pi) = 2$. Compute $f(x)$.

6. [10 Points] A ball is thrown upwards from the top of a building with an initial *speed* of 32 feet per second. The ball hits the ground below with a *speed* of 64 feet per second. How tall is the building?

Hint: Use $a(t) = -32$ feet per second squared as acceleration due to gravity on the falling body.