Tips for Physics Falling Bodies Problems

Equations of Vetical Motion

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Define $s(t)$ as the position of an object at time t.

Define $v(t)$ as the velocity at time t, the rate of change of position with respect to time.

Define $a(t)$ as the acceleration at time t, the rate of change of velocity with respect to time.

That means $v(t) = s'(t) = \dfrac{ds}{dt}$ is the **first** derivative of the position function.

That means $a(t) = v'(t) = \dfrac{dv}{dt} = s''(t)$ is the $\begin{cases} \textbf{first} \text{ derivative of the velocity function} \\ \textbf{second} \text{ derivative of the position function} \end{cases}$

$$\overrightarrow{\text{DIFFERENTIATE}}$$

$$s(t) \overset{\frac{d}{dt}}{\curvearrowright} v(t) \overset{\frac{d}{dt}}{\curvearrowright} a(t)$$

$$\overleftarrow{\int \ldots \, dt} \qquad \overleftarrow{\int dt}$$

$$\overleftarrow{ANTIDIFFERENTIATE}$$

Vertical Motion: Falling Body Problems

We will consider Vertical Motion here. On the Vertical Axis,

$$\textbf{FIX} \begin{cases} \uparrow^{+} \text{ the upwards direction as measurement in the positive } + \text{ direction} \\ \downarrow_{-} \text{ the downwards direction as the measurement in the negative } - \text{ direction} \end{cases}$$

We fix the Accceleration due to gravity as $a(t) = -32$ feet per second squared. Then we use Antidifferentiation to solve Initial-Valued Differential Equations to find the Veloctiy and Position formulas, as derived in class.

$a(t)$	$= -32$	feet per second^2
$v(t)$	$= -32t + v(0)$ $= -32t + v_0$	feet per second
$s(t)$	$= -16t^2 + v(0)t + s(0)$ $= -16t^2 + v_0t + s_0$	feet

$$\text{Note: } \begin{cases} v_0 = v(0) & \text{is called the Initial Velocity, meaning the Velocity at time } t = 0 \\ s_0 = s(0) & \text{is called the Initial Position, meaning the Position at time } t = 0 \end{cases}$$

Key Information

We typically fix the ground as Position $s(t) = 0$. There are three types of motion for an object moving vertically from a given initial height, which may even be the ground. The Object can be

- Dropped down from a fixed starting height (from rest) with no imposed initial velocity
  $\hookrightarrow v_0 = 0$

- Thrown straight Up with a given initial + velocity
  $\hookrightarrow v_0 =$ positive

- Thrown straight Down with a given initial $-$ velocity
  $\hookrightarrow v_0 =$ negative

Recall, for functions,

$$\begin{cases} f'(x) > 0 \text{ positive} & \Longrightarrow \quad f(x) \text{ is increasing} \nearrow \\ f'(x) < 0 \text{ negative} & \Longrightarrow \quad f(x) \text{ is decreasing} \searrow \end{cases}$$

In this setting, these size arguments translate as follows:

$$\begin{cases} \text{Velocity } v(t) = s'(t) > 0 \text{ positive} & \Longrightarrow \quad \text{Position } s(t) \text{ is increasing} \uparrow \\ \text{Velocity } v(t) = s'(t) < 0 \text{ negative} & \Longrightarrow \quad \text{Position } s(t) \text{ is decreasing} \downarrow \end{cases}$$

That is, if an object is thrown UP, then the Initial Velocity is **positive** because its position is increasing (moving further in the positive direction).

OR if an onject is thrown DOWN, with a given initial force, then the Initital Velocity is **negative** because its position is decreasing (moving further in the negative direction).

Finally, when the object hits the ground, there are two key features.

- First, typically the position at *impact* is set as $s(t) = 0$. Read the problem otherwise...

- Second, the velocity at *impact* may be computed as a value, but we know v_{impact} is **negative** because the object *falling*, moving DOWN, meaning the position is more in the negative direction on the fixed vertical axis.

Practice: $\begin{cases} \text{if the position is decreasing, then its derivative (velocity) is negative} \\ \text{if the position is increasing, then its derivative (velocity) is positive} \end{cases}$

Key Types of Computations:

1. If given an input time value, then you can often **plug in** the given time t into either $s(t), v(t)$ or $a(t)$ and solve for the output value of interest.

2. If given an output function value for $s(t), v(t)$ or $a(t)$, then you can often **set** the function equal to the given output and solve for the input t. This is typically harder to do, and might involve algebra (including the Quadratic Formula).

Some Typical Questions:

- When is the Maximum Height reached? That is, t_{max} which occurs when $v(t) = 0$.

$\hookrightarrow$ Solve the derived velocity $v(t) = 0$ for time t_{max}.

- What is the Maximum Height? That is, s_{max} which occurs when $v(t) = 0$ at time t_{max}.

$\hookrightarrow$ Solve $v(t) = 0$ for time t_{max} (as described just above here) and plug that into position $s(t)$ to find the Maximum position $s_{max} = s(t_{max})$

- What is the Initial Velocity? That is, $v_0 = v(0)$.

$\hookrightarrow$ Use the derived equation for velocity and some other given information and solve for the remaining unknown v_0 in the $v(t)$ equation.

- What is the Initial Height? That is, $s_0 = s(0)$.

$\hookrightarrow$ Use the derived equation for position and some other given information and solve for the remaining unknown s_0 in the $s(t)$ equation.

- When did the object *hit the ground*? That is, t_{impact} which occurs when $s(t) = 0$.

$\hookrightarrow$ Use the derived equation for position, and solve the equation $s(t) = 0$ for the input time t labelled t_{impact}.

Again, this may involve some factoring of a Quadratic Equation or use of the Quadratice Formula. If you do factor, go ahead and try to factor off the -16 constant in front of the lead t^2 term...

- What is the Velocity at Impact when the object *hits the ground*? That is, v_{impact} which occurs when $s(t) = 0$.

$\hookrightarrow$ Find the time of impact (as described just above here) and plug that time t_{impact} **into** the derived velocity equation. That is, solve $v_{impact} = v(t_{impact})$. It should be negative.

Final tip: Write down all the given information that is translated from the word problem. Also write down what info you're solving for, and that can often lead you to analyze a specific equation, which can make you think about which info you already know and which you can solve for.