Math 106 Final Examination May 7, 2018

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.

- You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{-\ln 5}$, or $e^{3\ln 3}$ should be simplified.

- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [25 Points] Compute each of the following derivatives.

(a) $g'(x)$, where $g(x) = (\cos x)^{3x}$. Simplify.

(b) $\dfrac{d}{dx} \ln\left(\dfrac{(5-x^2)^9 \sqrt{1+\tan x}}{e^{-\cos x}\cdot \ln x}\right)$ Do not simplify the final answer here.

(c) $f'\left(\dfrac{\pi}{6}\right)$ where $f(x) = \dfrac{1}{2\tan^2 x} + \cos^2 x + \sec(2x)$. Simplify.

(d) $f'(x)$, where $f(x) = \dfrac{1}{\sqrt{\ln x}} + \dfrac{1}{\ln\sqrt{x}}$. Do not simplify.

2. [8 Points] (a) Prove that $\dfrac{d}{dx}\ln x = \dfrac{1}{x}$.

(b) Let $y = 5^x$. Prove that $\dfrac{dy}{dx} = 5^x(\ln 5)$

3. [56 Points] Compute each of the following integrals.

(a) $\displaystyle\int \frac{1}{\sqrt{x}\sqrt{2+\sqrt{x}}}\,dx$ (b) Show that $\displaystyle\int_0^{\frac{\pi}{6}} \tan(2x)\,dx = \ln\sqrt{2}$ (c) $\displaystyle\int \frac{1}{e^x\left(1-e^{-x}\right)^{\frac{4}{3}}}\,dx$

(d) $\displaystyle\int_0^{10} 3-|x-1|\,dx$ (e) Show that $\displaystyle\int_{-1}^{2} \frac{x^3}{x^2-5}\,dx = \frac{3}{2} - \ln(32)$

(f) $\displaystyle\int_2^6 \frac{1}{x^2}\cos\left(\frac{\pi}{x}\right)\,dx$ (g) $\displaystyle\int_1^{e^3} \frac{\sqrt{4-\ln x}}{x}\,dx$

4. [20 Points] Consider the function given by

$$f(x) = \frac{e^{9x}}{9} + e^9 + \frac{e^9}{x} - \frac{1}{9e^x} + 9x^e + \frac{9}{e^{9x}} + \frac{e}{x^9} + \frac{e^x}{e^{9x}} + e^x\cdot e^{9x} + e^{9-x}$$

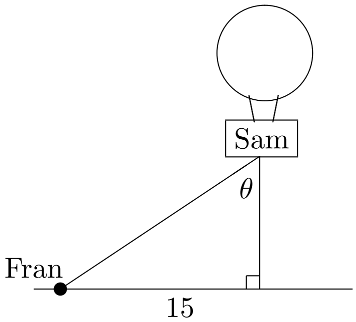
(a) Compute the **derivative**, $f'(x)$. (b) Compute the **antiderivative**, $\displaystyle\int f(x)\,dx$.

5. [15 Points] Compute $\displaystyle\int_{-1}^{2} 2-2x-x^2\,dx$ using each of the following **two** different methods:

(a) Fundamental Theorem of Calculus (b) The limit definition of the definite integral

Recall $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ and $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ and $\displaystyle\sum_{i=1}^{n} 1 = n$

6. [12 Points] Fran is standing on the ground 15 meters from a hot air balloon, which starts on the ground. The balloon starts floating straight up at 20 meters per minute. Sam is in the balloon basket watching Fran on the ground. Consider the angle θ, as shown in the diagram. At what rate is θ changing, when the balloon is 25 meters above the ground?



7. [14 Points] (a) At what **Point** on the curve $y = [(x-4)\cdot\ln(x-4)] - 4x + 16$ is the tangent line horizontal?

(b) Find the Absolute Maximum and/or Minimum **Values** for the function $f(x) = \dfrac{x^4}{e^x}$.

8. [15 Points] For each of parts (a), (b), (c), **Set-Up** but **DO NOT EVALUATE** the integral to compute the **AREA** bounded in the described regions. Sketch the region.

(a) The Region bounded by $y = 4 - x^2$, $y = x + 2$, between $x = -2$, and $x = 2$.

(b) The Region bounded by $y = \cos x$, $y = 1$, between $x = 0$, and $x = 2\pi$.

(c) The Region bounded by $y = \sin x$, $y = x$, between $x = 0$, and $x = \pi$. (Hint: what is the tangent line to $y = \sin x$ at $x = 0$?)

9. [15 Points] (a) Consider the region bounded by $y = e^x + 1$, $y = 3$, and $x = 0$. Sketch the bounded region. **COMPUTE** the integral to compute the Volume of the three-dimensional solid obtained by rotating the region about the horizontal line $y = -2$. Sketch the Solid of revolution, along with one Approximating Washer.

(b) Consider the region bounded by $y = \ln x$, $y = 1$, and $x = e^3$. Sketch the bounded region. **Set-up** but **DO NOT EVALUATE** the integral to compute the Volume of the three-dimensional solid obtained by rotating the region about the x-axis. Sketch the Solid of revolution, along with one Approximating Washer.

10. [10 Points] Jack throws a baseball straight downward from the top of a building. This initial *speed* of the ball is 25 feet per second. The ball hits the ground with a *speed* of 153 feet per second. How tall is the building? How far above the ground is the ball one second after Jack throws it down? (Hint: use acceleration $a(t) = -32$ feet per second squared.)

11. [10 Points] A population of bacteria was growing exponentially. Initially there were 3 cells. After 2 hours there were 9 cells. How many cells were there after 6 hours? When were there 123 cells?