Fundamental Theorem of Calculus

Fundamental Theorem of Calculus, Part 1

If f is continuous on $[a, b]$, then the function g defined by

$$g(x) = \int_a^x f(t)\ dt \quad a \leq x \leq b$$

is continuous on$[a, b]$ and differentiable on(a, b), and $g'(x) = f(x)$.
That is,

$$g'(x) = \frac{d}{dx} \int_a^x f(t)\ dt = f(x)$$

Think: if you have a Definite Integral with a constant a as a Lower Limit of Integration, and a variable (here as x) as the Upper Limit, then the Derivative of the *variable* Integral is equal to the Integrand f evaluated simply at x. More simply put, you get back the function inside the integral and plug in the single variable x. The order of the limits is important here.

Example:

If $g(x) = \displaystyle\int_3^x \sqrt{5 \cos t}\ dt$, then $g'(x) = \displaystyle\frac{d}{dx} \int_3^x \sqrt{5 + \cos t}\ dt = \boxed{\sqrt{5 + \cos x}}$

Fundamental Theorem of Calculus, Part 2

If f is continuous on $[a, b]$, then

$$\int_a^b f(x)\ dx = F(x)\Big|_a^b = F(b) - F(a)$$

where F is any Antiderivative of f, that is, a function F such that $F' = f$.

Example:

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 6 \cos x\ dx = 6 \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x\ dx = 6 \sin x \Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}} = 6 \left(\sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{6}\right) \right)$$

$$= 6\left(1 - \frac{1}{2}\right) = 6\left(\frac{1}{2}\right) = \boxed{3}$$

Properties of the Definite Integral

1. Empty Area: $\int_a^a f(x)\ dx = 0$

2. Limits Order: $\int_b^a f(x)\ dx = -\int_a^b f(x)\ dx$

3. Constant Rule: $\int_a^b constant\ dx = constant \cdot (b-a)$

4. Constant Multiple Rule: $\int_a^b cf(x)\ dx = c\int_a^b f(x)\ dx$ where c is a constant

5. Summation Rule: $\int_a^b f(x)+g(x)\ dx = \int_a^b f(x)\ dx + \int_a^b g(x)\ dx$

6. Difference Rule: $\int_a^b f(x)-g(x)\ dx = \int_a^b f(x)\ dx - \int_a^b g(x)\ dx$

7. Split Area Rule: $\int_a^b f(x)\ dx = \int_a^c f(x)\ dx + \int_c^b f(x)\ dx$ for c with $a < c < b$

IMPORTANT: There is **NO** immediate Integration Rule for Products or Quotients.

• You can use Algebra to simplify the integrand into simpler Power Rule or light Trig pieces.

OR

• You can use u-substitution for special combos where a certain chunk of the integrand can temporarily be hidden, as say u, *and* the derivative piece du is also found in the same problem.