Extra Riemann Sum, Definite Integral Example

Evaluate $\int_{-2}^{3} x^2-4x+3\ dx$ using Riemann Sums and the limit definition of the definite integral.

Here $f(x) = x^2 - 4x + 3, \quad a = -2, \quad b = 3, \quad \Delta x = \dfrac{b-a}{n} = \dfrac{3-(-2)}{n} = \dfrac{5}{n}$
and $x_i = a + x_i = -2 + i\left(\dfrac{5}{n}\right) = -2 + \dfrac{5i}{n}$.

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\begin{aligned}
\int_{-2}^{3} x^2 - 4x + 3\ dx &= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(-2+\frac{5i}{n}\right)\left(\frac{5}{n}\right) \\
&= \lim_{n\to\infty} \frac{5}{n}\sum_{i=1}^{n} \left(-2+\frac{5i}{n}\right)^2 - 4\left(-2+\frac{5i}{n}\right) + 3 \\
&= \lim_{n\to\infty} \frac{5}{n}\sum_{i=1}^{n} \left(4 - \frac{20i}{n} + \frac{25i^2}{n^2} + 8 - \frac{20i}{n} + 3\right) \\
&= \lim_{n\to\infty} \frac{5}{n}\sum_{i=1}^{n} \left(\frac{25i^2}{n^2} - \frac{40i}{n} + 15\right) \\
&= \lim_{n\to\infty} \frac{5}{n}\sum_{i=1}^{n} \frac{25i^2}{n^2} - \frac{5}{n}\sum_{i=1}^{n}\frac{40i}{n} + \frac{5}{n}\sum_{i=1}^{n} 15 \\
&= \lim_{n\to\infty} \frac{125}{n^3}\sum_{i=1}^{n} i^2 - \frac{200}{n^2}\sum_{i=1}^{n} i + \frac{75}{n}\sum_{i=1}^{n} 1 \\
&= \lim_{n\to\infty} \frac{125}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) - \frac{200}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{75}{n}(n) \\
&= \lim_{n\to\infty} \frac{125}{6}\left(\frac{n}{n}\right)\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right) - \frac{200}{2}\left(\frac{n}{n}\right)\left(\frac{n+1}{n}\right) + 75 \\
&= \lim_{n\to\infty} \frac{125}{6}(1)\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) - (100)(1)\left(1+\frac{1}{n}\right) + 75 \\
&= \frac{125}{6}(1)(1)(2) - (100)(1)(1) + 75 = \frac{125}{3} - 100 + 75 \\
&= \frac{125}{3} - 100 + 75 = \frac{125}{3} - 25 = \frac{125}{3} - \frac{75}{3} = \boxed{\frac{50}{3}}
\end{aligned}
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