Summer Academy, Calculus with Algebra, 2019

Worksheet 9, Saturday, July 6, 2019

1. Graph the function $y = \sin x$ on the interval $[0, 2\pi]$. Answer the following:

   (a) $\sin 0 =$

   (b) $\sin \pi =$

   (c) $\sin \dfrac{\pi}{2} =$

   (d) $\sin \dfrac{3\pi}{2} =$

   (e) $\sin 2\pi =$

2. Graph the function $y = \cos x$ on the interval $[0, 2\pi]$. Answer the following:

   (a) $\cos 0 =$

   (b) $\cos \pi =$

   (c) $\cos \dfrac{\pi}{2} =$

   (d) $\cos \dfrac{3\pi}{2} =$

   (e) $\cos 2\pi =$

3. Compute the following trig. values. Justify by showing the work on the Unit Circle.

   (a) Compute $\sin \dfrac{2\pi}{3} =$

   (b) Compute $\cos \dfrac{2\pi}{3} =$

   (c) Compute $\sin \dfrac{5\pi}{3} =$

   (d) Compute $\cos \dfrac{5\pi}{3} =$

   (e) Compute $\cos \dfrac{3\pi}{4} =$

   (f) Compute $\tan \dfrac{5\pi}{6} =$

   (g) Compute $\sin \dfrac{11\pi}{6} =$

FACT: The derivative of $\sin x$ is equal to $\cos x$. That is,

$$\boxed{\frac{d}{dx}\sin x = \cos x} \qquad \textbf{Memorize.}$$

FACT: The derivative of $\cos x$ is equal to $-\sin x$. That is,

$$\boxed{\frac{d}{dx}\cos x = -\sin x} \qquad \textbf{Memorize.}$$

4. For each function below, find the equation of the tangent line to the curve $f(x)$ at the given x-coordinate.

   (a) $f(x) = \sin x$ at $x = 0$.

   (b) $f(x) = \cos x$ at $x = \dfrac{\pi}{6}$.

   (c) $f(x) = \tan x$ at $x = \dfrac{\pi}{3}$.

5. Use the above facts and differentiaton rules to show that

$$\boxed{\frac{d}{dx}\tan x = \sec^2 x} \qquad \textbf{Memorize.}$$

6. Use the above facts and differentiaton rules to show that

$$\boxed{\frac{d}{dx}\sec x = \sec x \tan x} \qquad \textbf{Memorize.}$$

   Practice:

   $\dfrac{d}{dx}\sin(2x) = \cos(2x)2$

   $\dfrac{d}{dx}\sin^2 x = \dfrac{d}{dx}(\sin x)^2 = 2\sin x(\cos x)$

   $\dfrac{d}{dx}\sin^2(3x) = \dfrac{d}{dx}(\sin(3x)^2 = 2\sin(3x)(\cos(3x))3$

7. Let $W(x) = \cos^2(2x) + \tan(2x) + 3\sec x$. Compute $W'\left(\dfrac{\pi}{6}\right)$. Simplify your answer completely.

8. Compute $f'(x)$ where $f(x) = \cos^4(x^3 - 5)$. Simplify.

9. Compute $f'(x)$ where $f(x) = \dfrac{\cos(3x)}{\tan x + \sin(5x^2)}$. Do not simplify.

10. Compute the derivative $\dfrac{dy}{dx}$ for the curve $y^2 + \cos x = xy$.

Turn in your own solutions.