Summer Academy, Calculus with Algebra, 2019

Worksheet 8, Friday, July 5, 2019

1. Compute the equation of the line that is tangent to the curve $f(x) = \dfrac{1}{x-7}$ at the point where $x = 5$. **Also** draw the graph of the given function **and** the graph of this specific tangent line all on one sketch.

2. Compute the derivative of each of the following functions. For these problems, you do **not** need to simplify your derivative. You may use the quicker Differentiation Rules at this point, unless otherwise stated.

   (a) $f(x) = \dfrac{6}{7}x - x^{\frac{6}{7}} + \dfrac{1}{7x^6} - \dfrac{1}{6}$

   (b) $y = \left(\sqrt{x} + \dfrac{1}{x}\right)^9$

   (c) $y = \dfrac{1}{\sqrt{x^2 - 5x + 3}}$

   (d) $y = \left(\dfrac{1}{x^3} + 7x\right)^{\frac{5}{7}} \left(x^4 - \dfrac{1}{x^7}\right)^{-5}$

   (e) $y = \dfrac{1}{\left(\dfrac{1}{x^7} + \sqrt{x^6 - 7}\right)^{\frac{6}{7}}}$

   (f) $y = \sqrt{x} + \dfrac{1}{\sqrt{x}} + \dfrac{1}{1+\sqrt{x}} + \dfrac{1}{\sqrt{1+x}}$

   (g) $y = \sqrt{\dfrac{x+5}{5-x}}$

3. Find the equation of the tangent line to this curve $y = \sqrt{x + (x^2+1)^3}$ at the point where $x = 1$.

4. Find the equation of the tangent line to this curve $y = \dfrac{6x}{\sqrt{x^2+3}}$ at the point where $x = 1$.

5. Find **all** x-coordinates at which the graphs of the following functions have horizontal tangent lines. Please **simplify** your derivatives first. Why?

   (a) $f(x) = (7x-3)^4(5x+2)^6$

   (b) $w(t) = t^2(1-t)^6$

6. Compute the derivative of $f(x) = \sqrt{3 - 4x^2}$ **two** different ways:
   - First use the **limit definition of the derivative**.
   - Second use the Chain Rule.

7. Let $f(x)$ and $g(x)$ be differentiable functions with the following table of values:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	4	-3	2	7
2	-2	6	1	5
3	3	-2	-1	0

   Let

$$h(x) = f(x) \cdot g(x)$$

$$k(x) = \frac{g(x)}{f(x)}$$

$$P(x) = f(x) \cdot f(x)$$
$$Q(x) = f \circ g(x)$$

$$W(x) = g \circ g(x).$$

   Compute $h'(1)$, $k'(3)$, $P'(1)$, $Q'(2)$, and $W'(1)$.

   Note: this problem is testing whether you know your differentiation rules, especially in the case when you don't know the actual function's ($f(x)$ or $g(x)$) formula. To compute the derivative at one specific x-value, you just need the derivative information of each function piece *at* that specific x-value. You don't need to know the entire function's formula. Think about which derivative values are required in each problem. Write out the derivative carefully, and then plug in your specific x-value.

8. Compute $\dfrac{dy}{dx}$ if $x^3 + x^2 y^{\frac{3}{2}} = y^3 + 7$. Use implicit differentiation. Your answer should contain both x and y.

9. Find the equation of the tangent line to the curve $x^3 + x^2 y = 6 - 4y^2$ at the point $(1, 1)$. Use implicit differentiation.

Turn in your own solutions.