Summer Academy, Calculus with Algebra, 2019

Worksheet 7, Wednesday, July 3, 2019

- 1. Compute the derivative of $f(x) = \sqrt{x}$ two different ways:
 - First use the **limit definition of the derivative**.
 - Second use the Power Rule.

IMPORTANT: Now memorize
$$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$$

- 2. Compute the derivative of each of the following functions.
 - (a) $f(x) = \pi^2$ (b) $f(x) = \frac{3}{4}x^8$ (c) $f(x) = x^3 - 4x + 6$ (d) $f(x) = x^7 + 5x^2 - 7x + 9$ (e) $f(x) = \frac{1}{x^3}$ (f) $f(x) = -\frac{12}{x^5}$ (g) $f(x) = 5x^2 - \sqrt{x} + x^{\frac{5}{7}} + \frac{3}{x} - \frac{2}{x^2}$
- 3. For each of the problems compute the derivative of each of the following functions in **two** ways:

• FIRST simplify the function algebra and THEN use the Sum and Power Differentiation Rules.

• FIRST use the Product or Quotient Differentiation Rules and THEN simplify your answers and see if it matches your answer from above

(a)
$$f(x) = \sqrt{x} \cdot (x - 1)$$

(b) $f(x) = \frac{x^2 + 4x + 3}{\sqrt{x}}$
(c) $f(x) = \frac{\sqrt{x} + x}{x^2}$
(d) $f(x) = (x^5 + \sqrt{x}) \left(\frac{1}{x} - 3x^2\right)$
(e) $f(x) = \frac{\sqrt{x} - \frac{1}{x^8}}{x^7}$

- 4. Compute the equation of the line that is tangent to the curve $y = (3x^2 + 5)(2 4x)$ at the point where x = 1.
- 5. Simplify the expression $3(x+1)^2(1-2x)^4 + (x+1)^34(1-2x)^3(-2)$ Hint: Look for Common factors.
- 6. For later purposes we need to practice solving.
 - (a) Consider the equation $x^2 + 2xyy' = 3y 7y'$. Solve for y'.
 - (b) Consider the equation $3y^2 \frac{dy}{dx} 5x^3y = 4x + 7\frac{dy}{dx}$. Solve for $\frac{dy}{dx}$.
- 7. Solve the equation $\frac{2x+1}{\sqrt{x}} \frac{x\sqrt{x}}{x+2} = 0$

Hint: First, put the left hand side over a common denominator. Then remember that a fraction is zero when the numerator is zero.

- 8. Consider the function $f(x) = \frac{7x+3}{1-5x}$. Compute the derivative f'(x) in two different ways:
 - First compute the derivative using the **limit definition of the derivative**.
 - Second compute the derivative using the **Quotient Rule**.
- 9. Find **all** *x*-coordinates at which the graphs of the following functions have horizontal tangent lines. Please **simplify** your derivatives first. Why?

(a)
$$f(x) = (2x+3)(7-4x)$$

(b)
$$f(x) = x^{\frac{3}{2}} - 4x$$

(c) $f(x) = \frac{5 - 2x}{3x + 4}$

(d)
$$f(x) = \frac{2-x}{x^2+5}$$

Turn in your own solutions.