Summer Academy, Calculus with Algebra, 2019

Worksheet 6, Tuesday, July 2, 2010

1. Explain why the given function is discontinuous at the given value a. Sketch the graph of the function.

   (a) $f(x) = \dfrac{1}{x+2}$ with $a = -2$.

   (b) $f(x) = \begin{cases} \dfrac{1}{x+2} & \text{if } x \neq -2 \\ 1 & \text{if } x = -2 \end{cases}$ with $a = -2$

   (c) $f(x) = \begin{cases} \sqrt{x-2} & \text{if } x > 2 \\ -3 & \text{if } x = 2 \\ 2-x & \text{if } x < 2 \end{cases}$ with $a = 2$.

2. Design a piece-wise defined function (try to be creative) that has three discontinuities: (a) removable (b) infinite (c) jump. Mark your graph carefully.

3. Use the Intermediate Value Theorem to show there exists a root of the equation $4x^3 - 6x^2 + 3x - 2 = 0$ between $x = 1$ and $x = 2$.

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Definition: The **Derivative of a function** f **at a number** a, denoted by $f'(a)$, is given by

$$(*) \quad \boxed{f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

By the definition from class, this value is the slope of the tangent line **at** the given point $(a, f(a))$. This value captures the steepness of the curve **at** that point.

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4. Suppose that $f(x) = 5 - 6x + 4x^2$.

   (a) Compute $f'(1)$ using (*) above. (Here $a = 1$)

   (b) Write the **equation of the tangent line** to the curve $y = f(x)$ at the point where $x = 1$.

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If we replace a by a variable x above, we obtain the **derivative function** $f'(x)$ as

$$(**)\quad \boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

We will call this the *limit definition of the derivative.* Here $f'(x)$ is the function that takes in any value x and spits out the derivative at x. That is, the slope of the tangent line at the point $(x, f(x))$.

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5. For each of the following, find $f'(x)$ using the *limit definition of the derivative* (**).

   (a) $f(x) = x^3$

   (b) $f(x) = x^4$

   (c) $f(x) = \sqrt{x}$

   (d) $f(x) = \dfrac{1}{x}$

   (e) $f(x) = \dfrac{x+1}{x-1}$

   (f) $f(x) = \dfrac{1}{\sqrt{x}}$

   (g) $f(x) = \dfrac{7-4x}{2x-5}$

6. Find an equation of the tangent line to the graph of $y = G(x)$ at $x = 3$ if $G(3) = -3$ and $G'(3) = 4$.

7. (Challenge) Find the values of a and b that make f continuous everywhere.

$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \leq x < 3 \\ 2x - a + b & \text{if } x \geq 3 \end{cases}$$

Turn in solutions.