

**Worksheet 5, Saturday, June 29, 2019**

Evaluate each of the following limits. Please **justify** your answers. Be clear if the limit equals a value,  $+\infty$  or  $-\infty$ , or Does Not Exist.

Infinite Limits:

1.  $\lim_{x \rightarrow 4} \frac{x^2 - x - 12}{x^2 - 8x + 16}$

2.  $\lim_{x \rightarrow 7} \frac{-x}{(x - 7)^2}$

Limits at Infinity:

3.  $\lim_{x \rightarrow \infty} \frac{9x^4 - 5x^2 + 7}{5x^4 + 6x - 3}$

4.  $\lim_{x \rightarrow \infty} \frac{1 - x^3}{7x^3 + x^2 - 100}$

5.  $\lim_{x \rightarrow \infty} \frac{x^4 + 3x^2 + 6}{x^3 + 7}$

6.  $\lim_{x \rightarrow \infty} \frac{x^6 - x^3 + x}{x^7 + x^5 - 9}$

Continuity Questions:

7. State the definition for a function  $g(x)$  that is continuous at  $x = -7$ .
8. Sketch 4 different graphs that are each not continuous at  $x = 4$  for different reasons.
9. Consider the function  $f(x)$  that is continuous at  $x = 3$ . Assume that  $f(3) = 4$ .
  - (a) Write the *definition* for  $f(x)$  being continuous at  $x = 3$ .
  - (b) Discuss what you know about  $\lim_{x \rightarrow 3} f(x) = ??$ . Why? Be clear and justify with mathematical notation.

10. Suppose that  $f$  and  $g$  are functions, **and**

- $\lim_{x \rightarrow 3} f(x) = 9$
- $\lim_{x \rightarrow 7} g(x) = -6$
- $\lim_{x \rightarrow 4} f(x) = 7$
- $g(x)$  is continuous at  $x = 7$ .
- $f(x)$  is continuous at  $x = 4$ .

Evaluate the following quantities and fully **justify** your answers. Do not just put down a value:

- (a)  $f(4) =$
- (b)  $g(7) =$
- (c) Compute  $g \circ f(4) =$
- (d) Does  $f(3) = 9$ ? Why or why not? Use math notation.

11. Suppose that  $f$  and  $g$  are functions, **and**

- $\lim_{x \rightarrow 7} g(x) = 3$
- $\lim_{x \rightarrow 2} g(x) = 6$
- $f(3) = 2$
- $g(x)$  is continuous at  $x = 7$  and  $x = 2$ .
- $\lim_{x \rightarrow 3} f(x) = 5$

Evaluate the following quantities and fully **justify** your answers. Do not just put down a value:

- (a)  $g(7) =$
- (b) Compute  $g \circ f(3) =$
- (c) Compute  $f \circ g(7) =$
- (d) Is  $f(x)$  continuous at  $x = 3$ ? Why or why not? Use math notation.

**Turn in solutions.**