Summer Academy, Calculus with Algebra, 2019

## Worksheet 3, Thursday, June 27, 2019

1. Consider $f(x) = \sqrt{x-7}$. Compute $\dfrac{f(x+h)-f(x)}{h}$. Make sure you simplify enough to cancel the $h$ in the denominator.

**Please read the following carefully:**

Recall that $\lim_{x\to a^+} f(x) = L$ is the **Right Hand Limit**. We will denote this as **RHL**. It means you are studying the limiting value (if it exists) of the output function values as the input values approach *a from the right of a*. Also recall that $\lim_{x\to a^-} f(x) = L$ is the **Left Hand Limit**. We will denote this as **LHL**. It means you are studying the limiting value (if it exists) of the output function values as the input values approach *a from the left of a*.

We write $\lim_{x\to a} f(x) = L$ to represent the full **two-sided limit**. We have the following result.

Theorem: $\boxed{\lim_{x\to a} f(x) = L \text{ if and only if } \lim_{x\to a^+} f(x) = L \text{ and } \lim_{x\to a^-} f(x) = L.}$

That is, the two-sided limit exists at $a$ if and only if **both** the one-sided limits, from the right and from the left, exist and are equal. If RHL$\neq$ LHL then we say the two-sided limit Does Not Exist or **DNE**.

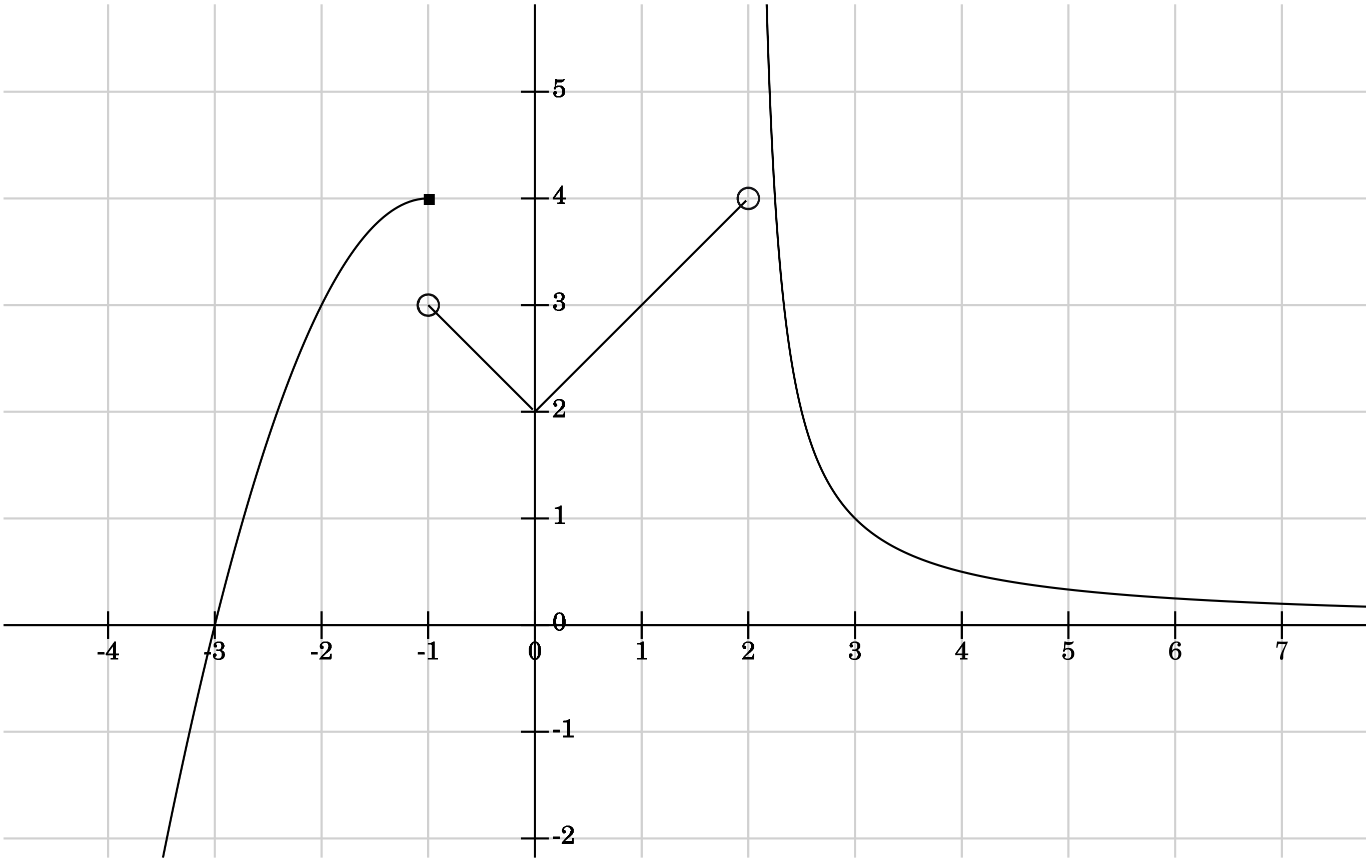
2. Sketch the graph of $f(x) = x^2$. Compute the following limits.

(a) $\lim_{x\to 0^+} f(x) =$ ________ (b) $\lim_{x\to 0^-} f(x) =$ ________ (c) $\lim_{x\to 0} f(x) =$ ________

3. Sketch the graph of $f(x) = 3-(x-1)^2$. Compute the following limits.

(a) $\lim_{x\to 1^+} f(x) =$ ________ (b) $\lim_{x\to 1^-} f(x) =$ ________ (c) $\lim_{x\to 1} f(x) =$ ________

4. Consider the following graph of $f(x)$. Answer the questions below. Justify your answers when necessary.




(a) $\lim_{x\to 2^+} f(x) =$ ________ (b) $\lim_{x\to 2^-} f(x) =$ ________ (c) $\lim_{x\to 2} f(x) =$ ________

(d) $\lim_{x\to -1^+} f(x) =$ ________ (e) $\lim_{x\to -1^-} f(x) =$ ________ (f) $\lim_{x\to -1} f(x) =$ ________

(g) $\lim_{x\to 0^+} f(x) =$ ________ (h) $\lim_{x\to 0^-} f(x) =$ ________ (i) $\lim_{x\to 0} f(x) =$ ________

(j) $\lim_{x\to \infty} f(x) =$ ________

(k) $\lim_{x\to -\infty} f(x) =$ ________

5. Consider the function defined piece-wise by $f(x) = \begin{cases} 2-x & \text{if } x < -1 \\ x+4 & \text{if } -1 < x \leq 1 \\ (x-2)^2 & \text{if } x > 1 \end{cases}$

Graph $f(x)$ carefully and find its Domain and Range. Compute each of the following, and justify when necessary.

(a) $\lim_{x \to 1^+} f(x) =$ ________ (b) $\lim_{x \to 1^-} f(x) =$ ________ (c) $\lim_{x \to 1} f(x) =$ ________

(d) $\lim_{x \to -1^+} f(x) =$ ________ (e) $\lim_{x \to -1^-} f(x) =$ ________ (f) $\lim_{x \to -1} f(x) =$ ________

6. Consider the function defined by

$$f(x) = \begin{cases} \sqrt{x-4} & \text{if } x > 4 \\ \\ 8-2x & \text{if } x < 4 \end{cases}$$

Graph $f(x)$ carefully and find its Domain and Range. Compute $\lim_{x \to 4} f(x)$, if it exists.
Hint: You might want to investigate one-sided limits on your own...

7. Consider the function defined by

$$f(x) = \begin{cases} x+1 & \text{if } x < -1 \\ \\ x^2 & \text{if } -1 \leq x < 1 \\ \\ 2-x & \text{if } x \geq 1 \end{cases}$$

Graph $f(x)$ carefully. Then determine the values $a$ for which $\lim_{x \to a} f(x)$ exists.

8. Compute the following limits. Justify your answers. Be clear if they equal a value, or $+\infty$, $-\infty$, or DNE.

(a) $\lim_{x \to 2} \dfrac{x^2 + 6x + 8}{x + 2}$

(b) $\lim_{x \to 2} \dfrac{x^2 - 6x + 8}{x - 2}$

(c) $\lim_{x \to -3} \dfrac{x^2 + 4x + 3}{x^2 - 2x - 15}$

(d) $\lim_{t \to 1} \dfrac{t - 1}{g(t^2) - 3}$ where $g(t) = 2t + 1$.

(e) $\lim_{x \to -5} \dfrac{\dfrac{1}{4 - x} - \dfrac{1}{9}}{x + 5}$

(f) $\lim_{x \to 4} \dfrac{|x - 4|}{x - 4}$ (challenge)

**Turn in your solutions**