Summer Academy, Calculus with Algebra, 2019

Worksheet 2, Wednesday, June 26, 2019

1. Write out the (cases) definition of $f(x) = |x+2|$. Sketch.

2. Write out the (cases) definition of $f(x) = |2x-1|$. Sketch.

3. Consider the function $f(x) = 1 + \frac{1}{x}$. Express $f(f(x))$ as a single fraction.

4. Consider the function defined piece-wise by $f(x) = \begin{cases} x+1 & \text{if } x \leq 0 \\ -x^2+6 & \text{if } x > 0 \end{cases}$

   Graph $f(x)$ and state its Domain and Range.

5. Consider the function defined piece-wise by $f(x) = \begin{cases} \frac{1}{x} & \text{if } x > 0 \\ x-3 & \text{if } -1 \leq x < 0 \\ -4 & \text{if } x < -1 \end{cases}$

   Graph $f(x)$ and state its Domain and Range.

6. Given two functions f and g. The **Composition** of f and g is defined by

$$f \circ g(x) = f(g(x))$$

   (a) Discuss what the Domain of $f \circ g$ is.

   (b) Take $f(x) = \sqrt{x+4}$ and $g(x) = x+2$. Compute **and** graph both $f \circ g$ and $g \circ f$. Discuss whether or not $f \circ g$ equals $g \circ f$. (Hint: what does it mean for two functions to be equal?)

7. Let $f(x) = \frac{x+1}{x-1}$. Compute $f(f(2))$. Compute and simplify $f(f(x))$. Hint: first find a large formula for $f(f(x))$. Then simplify by finding common denominators.

8. Let $f(x) = \frac{1}{x+1}$. Compute and simplify $\frac{f(x+h)-f(x)}{h}$.

   warning: $f(x+h) \neq f(x)+h$ **be careful!**

9. Let $f(x) = \dfrac{x-7}{x+3}$. Compute and simplify $\dfrac{f(x+h) - f(x)}{h}$.

10. Simplify each of the following expressions.

(a) $\dfrac{x^2 + 6x + 8}{x^2 - 4}$

(b) $\dfrac{x^2 + 6x + 8}{x^2 - 5x - 14}$

(c) $\dfrac{x^2 - 6x + 8}{x^2 - x - 2}$

(d) $\dfrac{1}{t\sqrt{1+t}} - \dfrac{1}{t}$

(e) $\dfrac{t-1}{g(t^2) - 3}$, where $g(t) = 2t + 1$

(f) $\dfrac{x^2 - 13x + 42}{x^2 - 4x - 12}$

(g) $\dfrac{1}{x} - \dfrac{1}{|x|}$ Hint: you might need two cases here. Write out the definition of $|x|$.

(h) $\dfrac{|x+4|}{x+4}$ Hint: you might need two cases here. Write out the definition of $|x+4|$.

(i) Let $f(x) = \dfrac{1}{x}$. Compute and simplify $\dfrac{f(t-1) - 2f(t)}{t^2 - 4}$

Turn in your solutions.