Summer Academy, Calculus with Algebra, 2019

Worksheet 1, Tuesday, June 25, 2019

1. Simplify each of the following expressions. Show your work. Justify

   (a) $\dfrac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)}$ (b) $\dfrac{1}{\left(\frac{a}{b}\right)}$

   (c) $\dfrac{\left(\frac{a}{b}\right)}{c}$ (d) $\dfrac{a}{\left(\frac{b}{c}\right)}$

2. Solve each of the following equations (if possible):

   (a) $x^2 - 4x - 21 = 0$ (b) $x^2 - x + 7 = 0$ (c) $x^2 + 2x - 4 = 0$

3. YES or NO: Does $\sqrt{x^2 + 4} = x + 2$? Why or why not?

4. Recall from class that we saw the graphs of $f(x) = \sqrt{x}$ and $g(x) = \dfrac{1}{x}$. Use these graphs to help you do the following:

   (a) Sketch the graph of $F(x) = \sqrt{x + 4}$. Discuss the Domain and Range for this new function.

   (b) Sketch the graph of $G(x) = \dfrac{1}{x - 6}$. Discuss the Domain and Range for this new function. Discuss the output behavior of $G(x)$ as the input value x is near $x = 6$. (Be specific.) Discuss the output behavior of $G(x)$ out near $\pm\infty$.

5. The Absolute Value Function $f(x) = |x|$ is a *piece-wise defined function* defined by

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

   (a) Give the Domain and Range for this function. Graph the absolute value function. Discuss how this function behaves near $x = 0$.

   (b) Now consider $g(x) = |x - 6|$. Write out the piece-wise defined definition of this function carefully. THEN use that definition to graph the function g. Discuss how this graph relates to the graph of $f(x) = |x|$. Discuss how this function behaves near $x = 6$.

   (c) Now consider $h(x) = |x + 7|$. Write out the piece-wise defined definition of this function carefully. THEN use that definition to graph the function h. Discuss how this graph relates to the graph of $f(x) = |x|$. Discuss how this function behaves near $x = -7$.

6. Find the equation of the line L that passes through the point $(3, -1)$ and is **perpendicular** to the line $2x + 5y = 6$. THEN, does this new line L pass through the point $(1, -6)$?

7. Consider the function $f(x) = x^2 - 6x - 7$. Compute and **simplify** each of the following.

   (a) $f(0)$

   (b) $f(-3)$

   (c) $f(1)$

   (d) For what values x does $f(x) = 0$?

   (e) $f(a)$

   (f) $f(a + h)$

   (g) $\dfrac{f(a+h) - f(a)}{h}$

   (h) CHALLENGE!!! Compute $f(f(x))$. Show that it equals $x^4 - 12x^3 + 16x^2 + 120x + 84$. Yes... simplify!

8. Consider the function defined piece-wise by

$$f(x) = \begin{cases} x + 2 & \text{if} \quad x > 2 \\ -3 & \text{if} \quad x = 2 \\ x^2 & \text{if} \quad -1 < x < 2 \\ 5 & \text{if} \quad x < -1 \end{cases}$$

   Graph $f(x)$ and find its Domain and Range.

9. Consider the function defined piece-wise by

$$g(x) = \begin{cases} \dfrac{1}{x} & \text{if} \quad x > 0 \\ -\dfrac{1}{2}x + 1 & \text{if} \quad -4 < x \leq 0 \\ x^2 & \text{if} \quad x \leq -4 \end{cases}$$

   Graph $g(x)$ and find its Domain and Range.

Each person turn in your own solutions.

You do need to understand ALL of these problems.

I will post answer keys on the class webpage.