Bonus Limits

Compute these extra limits if you have free time.

OPTIONAL BONUS #1

Compute $\lim_{x\to 2} \dfrac{(4-\sqrt{x+14})(\sqrt{13-x^2}-3)}{(6-\sqrt{40-2x})(\sqrt{x^2+21}-5)} =$

$= \lim_{x\to 2} \dfrac{(4-\sqrt{x+14})(\sqrt{13-x^2}-3)}{(6-\sqrt{40-2x})(\sqrt{x^2+21}-5)}$ why not throw in all the conjugates at once...

$$\cdot \left(\frac{4+\sqrt{x+14}}{4+\sqrt{x+14}}\right)\cdot\left(\frac{\sqrt{13-x^2}+3}{\sqrt{13-x^2}+3}\right)\cdot\left(\frac{6+\sqrt{40-2x}}{6+\sqrt{40-2x}}\right)\cdot\left(\frac{\sqrt{x^2+21}+5}{\sqrt{x^2+21}+5}\right)$$

$$= \lim_{x\to 2} \frac{(16-(x+14))((13-x^2)-9)(6+\sqrt{40-2x})(\sqrt{x^2+21}+5)}{(36-(40-2x))((x^2+21)-25)(4+\sqrt{x+14})(\sqrt{13-x^2}+3)}$$

$$= \lim_{x\to 2} \frac{(2-x)(4-x^2)(6+\sqrt{40-2x})(\sqrt{x^2+21}+5)}{(2x-4)(x^2-4)(4+\sqrt{x+14})(\sqrt{13-x^2}+3)}$$

$$= \lim_{x\to 2} \frac{-(x-2)(-1)(x^2-4)(6+\sqrt{40-2x})(\sqrt{x^2+21}+5)}{2(x-2)(x^2-4)(4+\sqrt{x+14})(\sqrt{13-x^2}+3)}$$

$$= \lim_{x\to 2} \frac{(6+\sqrt{40-2x})(\sqrt{x^2+21}+5)}{2(4+\sqrt{x+14})(\sqrt{13-x^2}+3)} = \frac{(6+\sqrt{36})(\sqrt{25}+5)}{2(4+\sqrt{16})(\sqrt{9}+3)} = \frac{(6+6)(5+5)}{2(4+4)(3+3)} = \boxed{\frac{5}{4}}$$

OPTIONAL BONUS #2

Compute $\lim_{x\to 0} \dfrac{|x-1|-|x+1|-|x|}{|x|+|2-x|-|x+2|} =$

DOES NOT EXIST, RHL $\neq$ LHL

RHL: $\lim_{x\to 0} \dfrac{|x-1|-|x+1|-|x|}{|x|+|2-x|-|x+2|} = \lim_{x\to 0^+} \dfrac{-(x-1)-(x+1)-x}{x+(2-x)-(x+2)}$

$= \lim_{x\to 0^+} \dfrac{-x+1-x-1-x}{x+2-x-x-2} = \lim_{x\to 0^+} \dfrac{-3x}{-x} = \boxed{3}$

LHL: $\lim_{x\to 0} \dfrac{|x-1|-|x+1|-|x|}{|x|+|2-x|-|x+2|} = \lim_{x\to 0^-} \dfrac{-(x-1)-(x+1)-(-x)}{-x+(2-x)-(x+2)} =$

$= \lim_{x\to 0^-} \dfrac{-x+1-x-1+x}{-x+2-x-x-2} = \lim_{x\to 0^-} \dfrac{-x}{-3x} = \boxed{\dfrac{1}{3}}$

Here, recall that $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

$$|x-1| = \begin{cases} x-1 & \text{if } x-1 \geq 0 \\ -(x-1) & \text{if } x-1 < 0 \end{cases} = \begin{cases} x-1 & \text{if } x \geq 1 \\ 1-x & \text{if } x < 1 \end{cases}$$

$$|x+1| = \begin{cases} x+1 & \text{if } x+1 \geq 0 \\ -(x+1) & \text{if } x+1 < 0 \end{cases} = \begin{cases} x+1 & \text{if } x \geq -1 \\ -x-1 & \text{if } x < -1 \end{cases}$$

$$|2-x| = \begin{cases} 2-x & \text{if } 2-x \geq 0 \\ -(2-x) & \text{if } 2-x < 0 \end{cases} = \begin{cases} 2-x & \text{if } x \leq 2 \\ x-2 & \text{if } x > 2 \end{cases}$$

$$|x+2| = \begin{cases} x+2 & \text{if} \quad x+2 \geq 0 \\ -(x+2) & \text{if} \quad x+2 < 0 \end{cases} = \begin{cases} x+2 & \text{if} \quad x \geq -2 \\ -x-2 & \text{if} \quad x < -2 \end{cases}$$

OPTIONAL BONUS #3 Compute $\lim\limits_{x\to 1} \dfrac{\dfrac{1}{\sqrt{2-x}} - \dfrac{2}{\sqrt{3+x}}}{\dfrac{7}{\sqrt{50-x}} - \dfrac{6}{\sqrt{x+35}}}$

$$= \lim_{x\to 1} \frac{\dfrac{\sqrt{3+x}-2\sqrt{2-x}}{\sqrt{2-x}\sqrt{3+x}}}{\dfrac{7\sqrt{x+35}-6\sqrt{50-x}}{\sqrt{50-x}\sqrt{x+35}}} = \lim_{x\to 1} \frac{\sqrt{50-x}\sqrt{x+35}\left(\sqrt{3+x}-2\sqrt{2-x}\right)}{\sqrt{2-x}\sqrt{3+x}\left(7\sqrt{x+35}-6\sqrt{50-x}\right)} \left(\tfrac{0}{0}\right)$$

$$= \lim_{x\to 1} \frac{\sqrt{50-x}\sqrt{x+35}\left(\sqrt{3+x}-2\sqrt{2-x}\right)}{\sqrt{2-x}\sqrt{3+x}\left(7\sqrt{x+35}-6\sqrt{50-x}\right)} \cdot \left(\frac{\sqrt{3+x}+2\sqrt{2-x}}{\sqrt{3+x}+2\sqrt{2-x}}\right) \qquad \text{one conjugate}$$

$$= \lim_{x\to 1} \frac{\sqrt{50-x}\sqrt{x+35}\left((3+x)-4(2-x)\right)}{\sqrt{2-x}\sqrt{3+x}\left(7\sqrt{x+35}-6\sqrt{50-x}\right)\left(\sqrt{3+x}+2\sqrt{2-x}\right)}$$

$$= \lim_{x\to 1} \frac{\sqrt{50-x}\sqrt{x+35}\,(5x-5)}{\sqrt{2-x}\sqrt{3+x}\left(7\sqrt{x+35}-6\sqrt{50-x}\right)\left(\sqrt{3+x}+2\sqrt{2-x}\right)} \cdot \left(\frac{7\sqrt{x+35}+6\sqrt{50-x}}{7\sqrt{x+35}+6\sqrt{50-x}}\right)$$

other conjugate

$$= \lim_{x\to 1} \frac{\sqrt{50-x}\sqrt{x+35}\,(5x-5)\left(7\sqrt{x+35}+6\sqrt{50-x}\right)}{\sqrt{2-x}\sqrt{3+x}\left(49(x+35)-36(50-x)\right)\left(\sqrt{3+x}+2\sqrt{2-x}\right)}$$

$$= \lim_{x\to 1} \frac{\sqrt{50-x}\sqrt{x+35}\,(5x-5)\left(7\sqrt{x+35}+6\sqrt{50-x}\right)}{\sqrt{2-x}\sqrt{3+x}\left(49x+1715-1800+36x\right)\left(\sqrt{3+x}+2\sqrt{2-x}\right)}$$

$$= \lim_{x\to 1} \frac{\sqrt{50-x}\sqrt{x+35}\,(5(x-1))\left(7\sqrt{x+35}+6\sqrt{50-x}\right)}{\sqrt{2-x}\sqrt{3+x}\,(85x-85)\left(\sqrt{3+x}+2\sqrt{2-x}\right)}$$

$$= \lim_{x\to 1} \frac{\sqrt{50-x}\sqrt{x+35}\,(5(x-1))\left(7\sqrt{x+35}+6\sqrt{50-x}\right)}{\sqrt{2-x}\sqrt{3+x}\,(85(x-1))\left(\sqrt{3+x}+2\sqrt{2-x}\right)}$$

$$= \lim_{x\to 1} \frac{\sqrt{50-x}\sqrt{x+35}\,(5)\left(7\sqrt{x+35}+6\sqrt{50-x}\right)}{\sqrt{2-x}\sqrt{3+x}\,(85)\left(\sqrt{3+x}+2\sqrt{2-x}\right)} \qquad \text{(x-1) factor finally cancels}$$

$$= \frac{7\cdot 6\cdot 5\cdot 84}{1\cdot 2\cdot 85\cdot 4} = \boxed{\frac{441}{17}}$$

OPTIONAL BONUS #4 Compute $\lim\limits_{x\to 1} \dfrac{2|x-1|-|x+2|+|x|+|x+1|}{|x-1|+|3-x|-|x+1|} =$