Summer Academy, Calculus with Algebra, 2019

Another Practice Exam #2, Friday, July 5, 2019

1. Explain why the given function is discontinuous at each of the given values a. Sketch the graph of the function.

$$f(x) = \begin{cases} \dfrac{1}{x-4} & \text{if } x > 4 \\ -x & \text{if } 0 < x < 4 \\ 3 & \text{if } x = 0 \\ -x^2 & \text{if } -2 < x < 0 \\ 3-(x+2)^2 & \text{if } x \leq -2 \end{cases} \qquad \text{with } a = -2,\ a = 0, \text{ and } a = 4.$$

2. Evaluate each of the following limits. Please **justify** your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

   (a) $\lim\limits_{x\to 7} \dfrac{x^2-6x-7}{x^2-14x+49} =$

   (b) $\lim\limits_{x\to\infty} \dfrac{4x^2+8x-1}{5x^2-7} =$

   (c) $\lim\limits_{x\to\infty} \dfrac{3x^6-5}{9x^4+3x} =$

   (d) $\lim\limits_{x\to\infty} \dfrac{2x^4-9x+8}{x^9+3} =$

3. Compute the derivative of each of the following functions. For these problems, you do **not** need to simplify your derivative. You may use the quicker Differentiation Rules at this point, unless otherwise stated.

   (a) $f(x) = \dfrac{5}{6}x + x^{\frac{5}{6}} + x^{\frac{6}{5}} + \dfrac{1}{x^{\frac{5}{6}}} + \dfrac{6}{5} + \dfrac{5}{6x^6} - \dfrac{6}{x^5}$

   (b) $y = \sqrt{\dfrac{x}{7} - \dfrac{1}{x^7}}$

   (c) $f(x) = \left(x^4 - \dfrac{4}{x^4}\right)\left(4x + \dfrac{\sqrt{x}}{4}\right)$

   (d) $y = \left(\dfrac{1}{3x^8} + 8x^3\right)^{\frac{3}{8}}$

4. Compute the derivative of $f(x) = \dfrac{6-5x}{2+4x}$ **two** different ways:

   • First use the **limit definition of the derivative**.

   • Second use the Quotient Rule.

   Next, simplify your answer in the first part. Then compute the second derivative $f''(x)$.

5. Find **all** x-coordinates at which the graphs of the following functions have horizontal tangent lines. Please **simplify** your derivatives first. Why?

   (a) $f(x) = \dfrac{(4x+3)^3}{(8x+2)^4}$

   (b) $f(x) = (x+1)^2 \cdot \sqrt{x+2}$

6. Compute the derivative of $f(x) = \dfrac{x}{x-1} + \dfrac{x}{x+1}$. Simplify your answer to a single fraction.

7. Find the equation of the tangent line to the curve $x^3 + x^2y = 6 - 4y^2$ at the point $(1, 1)$.