

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or webpages, or other aids are permitted.
- Please *show* all of your work and *justify* all of your answers.

1. Suppose that f and g are functions, **and**

- $\lim_{x \rightarrow 7} f(x) = 5$
- $f(5) = 7$
- $g(7) = -3$
- $g(x)$ is continuous at $x = 7$.
- $f(x)$ is NOT continuous at $x = 7$

Evaluate the following quantities and fully **justify** your answers. Do not just put down a value:

- $\lim_{x \rightarrow 7} g(x) =$
 - $g \circ f(5) =$
 - Does $f(7) = 5$? Justify.
2. Evaluate each of the following limits. Please **justify** your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

- $\lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x^2 - 6x + 9} =$
- $\lim_{x \rightarrow \infty} \frac{3x^2 - 5x + 6}{7 - 5x^2} =$
- $\lim_{x \rightarrow \infty} \frac{4x^2 - 5}{3x^7 + 9} =$
- $\lim_{x \rightarrow \infty} \frac{6x^3 - 4}{x + 5} =$

3. Compute the derivative of each of the following functions. For these problems, you do **NOT** need to simplify your derivative.

(a) $y = \frac{5}{6}x + x^{\frac{5}{6}} + \frac{1}{x^{\frac{5}{6}}} + \sqrt{5x + 6} + \frac{1}{\sqrt{5x + 6}}$

(b) $y = \left(\frac{2\sqrt{x} + x^3}{x^{\frac{2}{3}} + \frac{2}{3}x} \right)^{\frac{2}{3}}$

(c) $f(x) = \left(\frac{3}{x^2} - \frac{2}{x^3} \right)^9 \sqrt{5 - x^2}$

(d) $y = \frac{\frac{1}{x} - 6x^3}{\sqrt{7x + x^8}}$

4. Find the equation of the tangent line to this curve $y = \left(6x + \sqrt{8 + x^2}\right)^{\frac{3}{2}}$ at the point where $x = 1$.

5. Compute the derivative of $f(x) = \frac{3x - 1}{2 - 5x}$ **two** different ways:

- First use the **limit definition of the derivative**.
- Second use the Chain Rule.

Next simplify your answer from the first part. Then compute the second derivative $f''(x)$. Simplify your final answer to a single fraction.

6. Find **all** x -coordinates at which the graph of the following function has horizontal tangent lines. Please **simplify** your derivative first. Why?

$$f(x) = (5 + 3x^2)^8(7 - x^2)^3$$

7. Consider $f(x) = \frac{5x}{1 + x}$. Compute $f'(0)$, $f'(1)$, and $f'(2)$,

8. Find the equation of the tangent line to the curve $4(x + y)^2 = x^2y^2$ at the point $(-2, 1)$.