

Name: _____

Summer Academy
Midterm Exam #2
July 6, 2019

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or webpages, or other aids are permitted.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

Problem	Score	Possible Points
1		20
2		28
3		8
4		18
5		10
6		16
Total		100

1. [20 Points] Evaluate each of the following limits. Please **justify** your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

$$(a) \lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x^2 - 6x + 9} =$$

$$(b) \lim_{x \rightarrow \infty} \frac{3x^2 - 5x + 6}{7 - 5x^2} =$$

$$(c) \lim_{x \rightarrow \infty} \frac{4x^2 - 5}{3x^7 + 9} =$$

$$(d) \lim_{x \rightarrow \infty} \frac{6x^3 - 4}{x + 5} =$$

2. [28 Points] Compute the derivative of each of the following functions. For these problems, you do **NOT** need to simplify your derivative.

(a) $y = \frac{5}{6}x + x^{\frac{5}{6}} + \frac{1}{x^{\frac{5}{6}}} + \sqrt{5x+6} + \frac{1}{\sqrt{5x+6}}$

(b) $y = \left(\frac{2\sqrt{x} + x^3}{x^{\frac{2}{3}} + \frac{2}{3}x} \right)^{\frac{2}{3}}$

2. (Continued) Compute the derivative of each of the following functions. For these problems, you do **NOT** need to simplify your derivative.

(c) $f(x) = \left(\frac{3}{x^2} - \frac{2}{x^3} \right)^9 \sqrt{5 - x^2}$

(d) $y = \frac{\frac{1}{x} - 6x^3}{\sqrt{7x + x^8}}$

3. [8 Points] Find the equation of the tangent line to this curve $y = \left(6x + \sqrt{8 + x^2}\right)^{\frac{3}{2}}$ at the point where $x = 1$.

4. [18 Points]

(a) Compute the derivative of $f(x) = \frac{3x - 1}{2 - 5x}$ **two** different ways:

- First use the **limit definition of the derivative**.
- Second use the Quotient Rule.

4. (Continued)

(b) Compute the derivative of $f(x) = \sqrt{3 - x + x^2}$ **two** different ways:

- First use the **limit definition of the derivative**.
- Second use the Chain Rule.

5. [10 Points] Suppose that G and H are functions, **and**

- $\lim_{x \rightarrow 5} G(x) = 6$
- $\lim_{x \rightarrow -9} H(x) = -4$
- $\lim_{x \rightarrow 8} G(x) = 7$
- $G(x)$ is continuous at $x = 8$.
- $H(x)$ is continuous at $x = 7$.
- $G(5) = -9$
- $H(7) = -9$

Answer the following questions or evaluate the following quantities and fully **justify** your answers.

(a) Compute $G(8) =$

(b) Compute $\lim_{x \rightarrow 7} H(x) =$

(c) Compute $H \circ G(8) =$

(d) Does $H(-9) = -4$? Yes, No, or Not Enough Information? Why or why not?

(e) Is $G(x)$ continuous at $x = 5$?

6. [16 Points] Consider the function defined by

$$f(x) = \begin{cases} \sqrt{x-3} & \text{if } x > 3 \\ 5 & \text{if } x = 3 \\ 1 - (x-2)^2 & \text{if } 2 < x < 3 \\ 5 - 2x & \text{if } 0 \leq x < 2 \\ \frac{1}{x+3} & \text{if } x < 0 \end{cases}$$

(a) Carefully sketch the graph of $f(x)$.

(b) State the value(s) at which f is discontinuous. Justify your answer(s) using definitions or theorems discussed in class.

OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

OPTIONAL BONUS #1 Consider the equation $\frac{x}{y+1} = x^2 - y^2$. Compute the equation of the tangent line at the point $(1, 0)$.

OPTIONAL BONUS #2 Let $f(x) = \frac{x^2 + 1}{7 - x^3}$. Compute the Derivative $f'(x)$ using the Limit Definition of the Derivative. (You can check your answer by using the Quotient Rule).