- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.

- Please *show* all of your work and *justify* all of your answers.

1. Evaluate each of the following limits. Please **justify** your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\displaystyle\lim_{x\to-7} \frac{x^2 + 5x - 14}{x^2 - 4x + 4} =$

(b) $\displaystyle\lim_{x\to-6} \frac{f(x^2) + 5x - 8}{[f(x)]^2 + 5x + 14} = \qquad$ where $f(x) = x + 2$

(c) $\displaystyle\lim_{x\to8} \frac{3 - \sqrt{x + 1}}{x^2 - 7x - 8} =$

(d) $\displaystyle\lim_{x\to4} \frac{\dfrac{3 - x}{x - 5} - \dfrac{3}{7 - x}}{x^2 - x - 12} =$

(e) $\displaystyle\lim_{x\to7} \frac{x^2 - 4x - 21}{x^2 - 3x} =$

(f) $\displaystyle\lim_{x\to4} \frac{x^2 - 3x - 4}{|x - 4|} =$

(g) $\displaystyle\lim_{x\to4} \frac{x^2 - 2x - 8}{x^2 - 5x + 4} =$

(h) $\displaystyle\lim_{x\to-5} \frac{\dfrac{1}{1 - x} - \dfrac{1}{6}}{x^2 + 3x - 10} =$

(i) $\displaystyle\lim_{x\to3} \frac{x^2 - 12x + 27}{x^2 - 9} =$

(j) $\displaystyle\lim_{x\to2} \frac{\sqrt{x + 7} - 3}{x^2 - 3x + 2}$

2. Suppose that $f(x) = \dfrac{x+7}{x-3}$. Compute the difference quotient $\dfrac{f(x+h) - f(x)}{h}$. Simplify your answer until the h in the denominator cancels.

3. Consider the two functions $f(x) = \dfrac{1+x}{1-x}$ and $g(x) = \dfrac{1}{x}$.

(a) Compute $f \circ g(x)$. Simplify your answer to a single fraction. State the Domain.

(b) Compute $g \circ f(x)$. Simplify your answer to a single fraction. State the Domain.

(c) Compute $f \circ f(x)$. Simplify your answer to a single fraction. State the Domain.

4. Consider the function defined by

$$f(x) = \begin{cases} 2 & \text{if } x \geq 11 \\ \sqrt{x-7} & \text{if } 7 < x < 11 \\ 1 & \text{if } x = 7 \\ 7 - x & \text{if } 0 < x < 7 \\ 16 - x^2 & \text{if } -4 < x \leq 0 \\ \dfrac{1}{x+4} & \text{if } x < -4 \end{cases}$$

(a) Carefully sketch the graph of $f(x)$.

(b) State the Domain of the function $f(x)$.

(c) Compute $\lim\limits_{x \to -4} f(x) =$

(d) Compute $\lim\limits_{x \to 0} f(x) =$

(e) Compute $\lim\limits_{x \to 7} f(x) =$

(f) Compute $\lim\limits_{x \to 11} f(x) =$

(g) Compute $\lim\limits_{x \to \infty} f(x) =$

(h) Compute $\lim\limits_{x \to -\infty} f(x) =$

5. Consider the function defined by

$$f(x) = \begin{cases} \dfrac{1}{x-2} & \text{if } x > 2 \\[2mm] 4 - x^2 & \text{if } 0 \le x < 2 \\[2mm] x + 4 & \text{if } -6 < x < 0 \\[2mm] 1 - (x+6)^2 & \text{if } x \le -6 \end{cases}$$

Graph $f(x)$.

Answer the following questions. Justify your answers.

(a) $\lim\limits_{x \to -6} f(x) =$

(b) $\lim\limits_{x \to 0} f(x) =$

(c) $\lim\limits_{x \to 2} f(x) =$

(d) $\lim\limits_{x \to \infty} f(x) =$

(e) $\lim\limits_{x \to -\infty} f(x) =$