

Name: _____

Summer Academy
Midterm Exam #1
June 30, 2019

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.

- Simplify your answers.

- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

Problem	Score	Possible Points
1		36
2		8
3		16
4		10
5		20
Total		90

1. [36 Points] Evaluate each of the following limits. Please **justify** your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x-3} =$

(b) $\lim_{x \rightarrow 5} \frac{x^2 - 3x - 10}{|x - 5|} =$

(c) $\lim_{x \rightarrow 1} \frac{x^2 - 5x + 3}{x^2 - 2x} =$

1. (Continued) Evaluate each of the following limits. Please **justify** your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

$$(d) \lim_{x \rightarrow 3} \frac{\frac{2}{x-1} - \frac{3}{x}}{x-3} =$$

$$(e) \lim_{x \rightarrow -3} \frac{G(x^2) - x - 9}{G(x+6) + x^2 + x - 6} = \quad \text{where } G(x) = x - 3.$$

$$(f) \lim_{x \rightarrow 2} \frac{x^2 - 7x + 10}{x^2 + 2x - 8} =$$

2. [8 Points] Suppose that $f(x) = \frac{1}{x^2}$. Compute the difference quotient $\frac{f(x+h) - f(x)}{h}$. Simplify your answer until the h in the denominator cancels.

3. [16 Points] Consider the two functions $f(x) = \frac{1}{x}$ and $g(x) = x - 5$. Compute each of the following. Simplify your answers. **THEN** Sketch each graph.

(a) $f(x + 3) =$

(b) $f(x^2) + 3 =$

(c) $g(x^2) =$

(d) $[g(x)]^2 =$

(e) $g \circ f(x) =$

(f) $f \circ f(x) =$

(g) $f \circ g(x) =$

(h) $g \circ g(x) =$

4. [10 Points] For each of the following problems below, sketch any graph for the function f with the description given.

(a) Sketch a graph of any function f for which $\lim_{x \rightarrow 2} f(x)$ **Exists**.

(b) Sketch a graph of any function f for which $\lim_{x \rightarrow 2} f(x) = 5$.

(c) Sketch a graph of any function f for which $\lim_{x \rightarrow 2} f(x)$ **Does not Exist**.

(d) Sketch a graph of any function f for which $\lim_{x \rightarrow 2} f(x) = -1$ and $f(2) = 4$.

(e) Sketch a graph of any function f for which $\lim_{x \rightarrow 2} f(x) = -\infty$ and $f(2)$ is undefined.

5. [20 Points] Consider the function defined by

$$f(x) = \begin{cases} \frac{1}{x-3} & \text{if } x > 3 \\ x^2 + 1 & \text{if } 0 < x < 3 \\ -2 & \text{if } x = 0 \\ x + 1 & \text{if } -2 < x < 0 \\ 5 - (x + 2)^2 & \text{if } x < -2 \end{cases}$$

(a) Carefully sketch the graph of $f(x)$.

(b) State the **Domain** of the function $f(x)$.

5. (Continued) Continue to consider the function defined by

$$f(x) = \begin{cases} \frac{1}{x-3} & \text{if } x > 3 \\ x^2 + 1 & \text{if } 0 < x < 3 \\ -2 & \text{if } x = 0 \\ x + 1 & \text{if } -2 < x < 0 \\ 5 - (x + 2)^2 & \text{if } x < -2 \end{cases}$$

(c) Compute $\begin{cases} \lim_{x \rightarrow -2^+} f(x) = \\ \lim_{x \rightarrow -2^-} f(x) = \\ \lim_{x \rightarrow -2} f(x) = \end{cases}$

(d) Compute $\begin{cases} \lim_{x \rightarrow 0^+} f(x) = \\ \lim_{x \rightarrow 0^-} f(x) = \\ \lim_{x \rightarrow 0} f(x) = \end{cases}$

(e) Compute $\begin{cases} \lim_{x \rightarrow 3^+} f(x) = \\ \lim_{x \rightarrow 3^-} f(x) = \\ \lim_{x \rightarrow 3} f(x) = \end{cases}$

(f) Compute $\lim_{x \rightarrow \infty} f(x) =$

(g) Compute $\lim_{x \rightarrow -\infty} f(x) =$